Math Power

Name: (First)_____(Last)_____

School: _____ Grade:_____

PSAT

Warm-ups

1. Gene and Joan each bought some Ink-O pens and ink eraser. Gene paid $1.75 for 3 pens and 1 eraser. Joan paid $1.25 for 2 pens and 1 erase. What is the price of one of the pens?

2. Simplify $(x + 1)(y + 1) - (x - 1)(y - 1)$.

3. The statement $x \Rightarrow y$ is defined to be true if $x/3 > y/2$; otherwise, $x \Rightarrow y$ is false. Which of the following is true?
   A) $3 \Rightarrow 2$
   B) $9 \Rightarrow 10$
   C) $12 \Rightarrow 6$
   D) $15 \Rightarrow 12$

4. A classroom has R rows desks with D desks in each row. On a particular day when all pupils are present 3 seats are left vacant. How many pupils are there?

5. How many quarters equal in value of $5x - 10$ dimes?

6. The dimension of a rectangular field is 120 yd by 80 yd. A 10-yard wide sidewalk is to be built around the field. What is the area in square yard of the sidewalk (shaded region)?
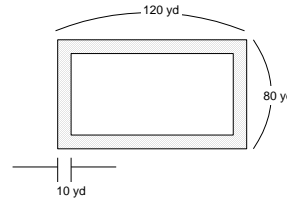
7. The sum of $2k$ and $k + 1$ is greater than 10 but less than 19. If k is an integer, what is one possible value of k?

8. If $0 < x < 1$, then which of the following must be true?
   A) $x^2 < 0$
   B) $x^2 > 1$
   C) $x^2 < x$
   D) $x^2 = x$

9. If $x - 1$ is a multiple of 3, write down
   (a) the next greater multiple of 3,

   (b) the previous smaller multiple of 3.

10. There are 120 red marbles and 80 blue marbles in a bag. How many blue marbles must be added so that the chances of getting a blue one from the bag are $\frac{2}{3}$?

Question set [11 - 13]

The chart shows below the number and types of chocolates in a box of candy.

Chocolate	Nuts	Fruits	Plain
Light	5	3	7
Dark	7	2	6

If a selection is made at random, find each of the following.

11. P(dark or nut)

12. P(light or fruit)

13. P(dark or plain)

14. 9 is $\frac{1}{3}$% of what number?

15. 75% of 20% of 400 is equal to which of the following?
    A) 6% of 75
    B) 30% of 75
    C) 60% of 75
    D) 80% of 75

16. Big Saving Supermarket reported a 20% increase customers each year. This year it has 6000 customers. How many more customers will it have next year than the previous year?

17. Ollie's Appliance Store reported 10% more customers this year than last. Last year Ollie had 7000 customers. How many did Ollie have this year?

18. A retailer has some sweaters that cost $25 each. It is priced to ensure a margin of 30% profit of the cost. If the sales tax is 6%, how much will a sweater cost after tax?

19. How can two families divide the 240 peaches so that one of the families has four times as many as the other one?

20. It would take Barb 4 hours to cut the lawn. Her mother could do it in 2 hours. How long would it take them both working together?

Warm-ups

21. Each available car for rent is red, blue, green or black. If there are between 70 and 150 cars available and exactly $\frac{1}{4}$ of the cars are red, exactly $\frac{1}{5}$ are blue, exactly $\frac{1}{6}$ are green, and the rest is black, what is a possible number of cars available?

22. Given $7x - 5y = 13$ and $2x - 7y = 26$. Find the value of $10x + 4y$.

PSAT

23. Solve: $\frac{(3x-1)}{4} = \frac{20}{3x+1}$

24. Solve for x:
$ax + c = bx + d, a \neq b$

25. If the sum of two numbers is 2 and their difference is 1, what is their product?

26. If $j^3 = \frac{1}{27}$ and $k^3 = \frac{1}{8}$, what is the value of $\frac{k}{j}$?
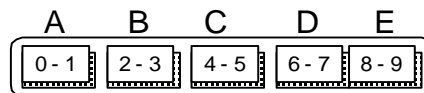
27. Express 4000π in scientific notation:
$a \times 10^b$
(Use 3.14 for π)

28. Simplify the rational expression:
$\left(\frac{x^9 y^6}{x^4 y^7}\right)^{-2}$
(Hint: Avoid negative exponent.)

29. How many hours are in $10a$ minutes?

Question set [**30 - 31**]

A display of the five buttons by the door handle of a car is as follows.

A	B	C	D	E
0-1	2-3	4-5	6-7	8-9

You can choose a 4-digit number as your password. For example, if your password is 4321, then you should press CBBA.

30. If repetition is allowed, how many different possible passwords can be chosen based on the numbers alone?

31. However, the car door cannot distinguish between 4321 and 5331 since the buttons pressed are both CBBA. As such, how many different passwords can the door lock recognize (repetition is permitted)?

32. A certain building has 2,600 square feet of surface that needs to be painted. If 1 gallon of paint will cover 250 square feet, what is the least whole number of gallons that must be purchased in order to have enough paint to apply one coat to the surface? (Assume that only whole gallons of paint can be purchased.)

33.
```
  AB
  A6
  A7
 +A9
 ────
 146
```
In the correct worked addition problem above, **A** and **B** stand for different digits. Find them.

Drafted by www.MathEnglish.com

34. 220% of what number is 66?

35. On a sale, Byron got a pair of roller blade with 20% discount for $120.
    (a) What is the discount he got?

    (b) How much did it cost originally?

36. After the first day registration, 840 enrolled students make the day school 30% full. How many more children can be admitted to the school?

37. Kim pays $630 after tax for a table. If the tax rate is 5%, what is the cost of the table before tax?

38. If 5 apples cost $1.25, how much will 16 apples cost?

39. If an object travels at x yards per minute, how many miles does it travel in y hours? (Hint: 1 yard = 3 ft, 1 mi = 5280 ft)

40. A car averages 20 miles per gallon of gas in city driving and 30 miles per gallon in highway driving. At these rates, if 15 gallons of gas were used in a 400-mile trip, how many miles are used in city driving?

Warm-ups

41. Solve the equation:
$$\frac{6}{x-1} - \frac{1}{x-2} = \frac{10}{2x-1}$$

42. If $(n + 3)(\frac{1}{5} - \frac{1}{9}) = 16$, then $n =$

42. If $(n + 3)(\frac{1}{5} - \frac{1}{9}) = 16$, then $n =$

43. If $x^2 - y^2 = 10$ and $x + y = 2.5$, what is the value of $y - x$?

44. For a number w, let
    ◄ w ► $= -w^2(w - 1)$.
    What is the value of ◄ (◄ -1 ►) ► ?

45. If $4x - 1 = 2\pi$, what is the value of $6x$ (in terms of π)?

PSAT

46. The weights of the members of the backfield of the Adams High School are 184 lb, 178 lb, 191 lb, and 167 lb. Bill Walters weighs 183 lb. Compared to the average weight of the back fielders, Bill's weight is
    A) 3 lb above the average
    B) 5 lb above the average
    C) 3 lb below the average
    D) 5 lb below the average

47. The average age of Alice, Brenda, Celia, Dana, Ellen is 12 years old. If the average age of Dana and Ellen is 9 years old. What is the average age of Alice, Brenda and Celia?

48. How many thirds are there in $\frac{3}{4}$?

49. Which of the following has the smallest value?
    A) $\frac{.1}{2}$
    B) $\frac{1}{.2}$
    C) $\frac{.1}{.2}$
    D) $\frac{.2}{.1}$

50. Which of the following has the smallest value?
    A) $\frac{5}{8}$
    B) $\frac{7}{12}$
    C) $\frac{18}{37}$
    D) $\frac{8}{15}$

51. Two hot dogs and a soda cost $3.25. If three hot dogs and a soda cost $4.50, what is the cost of two sodas?

52. In the figure, points P and T lie on line ℓ.

   How many different points on ℓ are twice as far from point T as from point P?
    A) None
    B) One
    C) Two
    D) Four

53. This year is a leap year. If July 1 of this year falls on a Saturday, then July 1 of next year falls on what day of the week?

54. A dealer sold an overcoat for $91. This was at a profit of 30% of the cost. What was the cost of the overcoat?

55. A price increased from $145 to $174, what was the percent increase?

56. Ms. Shapiro borrows $500 at interest rate of 15% annually. How much does she need to pay back at the end of 4 months?

57. After a discount of 30%, a purse was sold for $21.63. What was the original price?

58. Sharon went to visit her grandparents for the weekend, 50 miles away. The trip took her 1 hour to get there and $1\frac{1}{2}$ hours to get home. What was the average speed for the round trip?

59. It costs €135 to rent one square yard of space in Paris. At this rate, what is the rental cost in US dollars ($) per square foot? Assume that currencies exchange at the rate: 1 US dollar ($) = 0.75 Euro (€).

60. If the ratio of q to r is 4 to 5, which of the following could be true?
    A) $q = 0, r = 4/5$
    B) $q = 2, r = 5/2$
    C) $q = 5, r = 6$
    D) $q = 15, r = 12$

Real Challenge

61. In a batch of 10 light bulbs, there are two defective. If three bulbs are randomly picked (with replacement), what is the probability of the getting at least one defective?
    (A) $\frac{8}{15}$ (B) $\frac{7}{15}$ (C) 0.512 (D) 0.488 (E) $\frac{5}{8}$

62. The number 0.1234 is between $\frac{2n-1}{1000}$ and $\frac{2n}{1000}$ for some positive integer of n. Find the value of n.

63. In square ABCD below, AB = 2, and E, F, G and H are midpoints of the sides of the square.
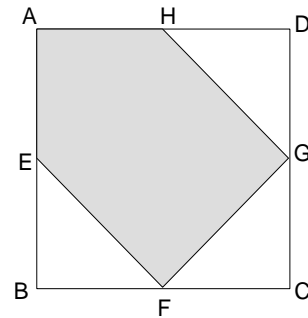
    What is the area of the shaded region?

64. If $f(x) = x^2 - 6x - 3.78$, what is the value of
    $f(7) - f(3)$?

65. If $2^{4x-4} = 16^5$, what is the value of $2x$?
    (A) 2.25 (B) 4.50 (C) 5.50 (D) 6.00 (E) 12.00

66. One square has a side 4 inches longer than the side of a second square. If the area of the larger square is 81 square inches, find the length of a side of each square.

67. Joy is packing for a trip. Of the towels in the closet, 6 are brown. She will randomly pick one of the towels to pack. If the probability of getting a brown towel is $\frac{2}{5}$, how many towels are in the closet?

68. Given $\frac{(x+3)^2}{3} + \frac{(y+4)^2}{4} + \frac{(z-5)^2}{5} = 0$, find the value for $x + y + z$.

69. Write a huge number with the positive integers 1, 2, 3, and so on integers in the number repetitions of itself.
    122333444455555…
    As it appears, the first digit is 1 and the 10th digit is 4. Also, the first occurrence of '5' is 11, while the last occurrence of '5' is 15.

    What is the 100th digit of this huge number?

70. In the number line below, the coordinates of P and T as shown. Find the coordinate of S that is twice as far from point T as from point P.
    (Hint: There are two possible locations for Q.)

    P(1)   T(7)

Drafted by www.MathEnglish.com

PSAT

Answer Key

1. Let p and e be the number of pens and erasers, respectively.
   $$\begin{cases} 3p + e = 1.75 \\ 2p + e = 1.25 \end{cases},$$
   $p = \boxed{\$0.50}$

2. $2(x + y)$

3. C
   $12 \div 3 = 4, 6 \div 2 = 3$
   $4 > 3$, so $12 \Rightarrow 6$ is true.

4. RD - 3

5. quarter : dime = 5:2 (value ratio)
   $5 \times \square = 2(5x - 10)$
   $\square = (5x - 10) \times \frac{2}{5} = \boxed{2x \text{ - } 4 \text{ (quarters)}}$

6. $120 \times 80 = 9600$ (outer rectangle)
   $100 \times 60 = 6000$ (inner rectangle)
   $9600 - 6000 = \boxed{3600 \text{ sq yd}}$

7. $10 < 2k + k + 1 < 19$
   $10 < 3k + 1 < 19$
   $9 < 3k < 18$
   $3 < k < 6$
   $k = \boxed{4 \text{ or } 5}$

8. C
   Assume $x = 0.1$, then $x^2 = 0.01 < 0.1 = x$.

9. (a) $x - 1 + 3 = x + 2$
   (b) $x - 1 - 3 = x - 4$

10. $120 + 80 = 200$ (total)
    Let x blues be added.
    $\frac{80+x}{200+x} = \frac{2}{3}$
    $240 + 3x = 400 + 2x$
    $x = \boxed{160}$

11. $\frac{2}{3}$

12. light or fruits = $5 + 3 + 7 + 2 = 15 + 2 = $ **17**

Chocolate	Nuts	Fruits	Plain
Light	5	3	7
Dark	7	2	6

    The probability is $\frac{17}{30}$.

13. $\frac{22}{30} = \frac{11}{15}$

14. $9 = \frac{1}{3}\% \times \boxed{2700}$

15. D
    $75\% \times 20\%\% \times 400$
    $= 4 \times 20\%\% \times 75$
    $= 80\%\% \times 75$

16. $6000 \times 1.2 = 7200$
    $6000 \div 1.2 = 5000$
    $7200 - 5000 = \boxed{2200}$

17. $1 + 10\% = 110\% = 1.1$
    $7000 \times 1.1 = \boxed{7700}$

18. $25 \times 1.3 \times 1.06 = \boxed{\$34.45}$

19. Let one family receive p and the other, $4p$ peaches.
    $p + 4p = 240$
    $5p = 240$
    $p = \boxed{48}$ and $4p = \boxed{192}$

20. It will be easier to view the problem in "the portion done in an hour".

	Job done in an hour
Barb	$\frac{1}{4}$
Mom	$\frac{1}{2}$
Bar + Mom	$\frac{1}{4} + \frac{1}{2} = \frac{3}{4}$

    Since they can finish $\frac{3}{4}$ of the job in an hour, they need
    $\frac{1}{\frac{3}{4}} = \frac{4}{3}$ hr = $1\frac{1}{3}$ hr = $\boxed{1 \text{ hr and } 20 \text{ min}}$

21. The least common multiple of 4, 5 and 6 is 60.
    $60 \times 2 = \boxed{120 \text{ cars}}$

22. Subtract the second equation from the first one.
    $2(5x + 2y) = \boxed{\text{-}26}$

23. $(3x - 1)(3x + 1) = 80$
    $9x^2 - 1 = 80$
    $9x^2 = 81$
    $x^2 = 9$
    $x = \boxed{\pm 3}$

24. $\frac{d-c}{a-b}$

25. $x + y = 2$
    $x - y = 1$
    $x = 1.5$
    $y = 0.5$
    $xy = 1.5 \times 0.5 = 0.75$

26. $j = \frac{1}{3}$ and $k = \frac{1}{2}$. Thus, $\frac{k}{j} = 1.5$

27. $4000\pi = 12560 = \boxed{1.256 \times 10^4}$

28. $\frac{y^2}{x^{10}}$

29. $10a \div 60 = \boxed{\frac{1}{6}a}$

30. $10 \times 10 \times 10 \times 10 = 10,000$

31. The trick is to focus on the button only. There are 5 buttons, therefore there are $5 \times 5 \times 5 \times 5 = \boxed{625}$ different button sequences.

32. $250×10 = 2500 < 2600$
$250× \boxed{11 \text{ gal}} = 2750$

33. **A** cannot be 2, neither 4.
A = $\boxed{3}$
B + 6 + 7 + 9 = $\boxed{26}$
B = $\boxed{4}$

34. $220\%·x = 66$
$x = 30$

35. (a) $120× \frac{1}{4} = \$30$
(b) $120÷0.8 = \$150$

36. $840× \frac{7}{3} = 1960$

37. $630÷1.05 = \$600$

38. $1.25÷5 = 0.25$
$16×0.25 = \boxed{\$4.00}$

39. $3x×60×y÷5280 = \boxed{\frac{3xy}{88}}$

40. Let the car run x gal in city, so it runs 15 - x on highway.
$20x + 30(15 - x) = 400$
$450 - 10x = 400$
$10x = 50$
$x = 5$ (mi in city)
$5×20 = \boxed{100 \text{ (mi)}}$

41. $\frac{6}{x-1} - \frac{1}{x-2} = \frac{5x-11}{(x-1)(x-2)}$
$\frac{\frac{5x-11}{(x-1)(x-2)}}{} = \frac{10}{2x-1}$
$(5x - 11)(2x - 1) = 10(x - 1)(x - 2)$
$10x^2 - 27x + 11 = 10x^2 - 30x + 20$
$3x = 9$
$x = \boxed{3}$

42. $(n + 3)4 = 16×45$
$n + 3 = 4×45$
$n = 177$

43. $x^2 - y^2 = (x + y)(x - y) = 2.5(x - y) = 10$
$x - y = 4$
$y - x = \boxed{-4}$

44. ◄ -1 ► = 2
◄ 2 ► = -4(1) = $\boxed{-4}$

45. $4x = 2π + 1$
$2x = π + \frac{1}{2}$
$6x = \boxed{3π + \frac{3}{2}}$

46. A
Method (I)
183 - 184 = -1
183 - 178 = 5
183 - 191 = -8
183 - 167 = 16
The sum of all the differences is 12. The average is $\frac{1}{4}(12) = 3$ (above)
Method (II)
The average weigh to the backfielders
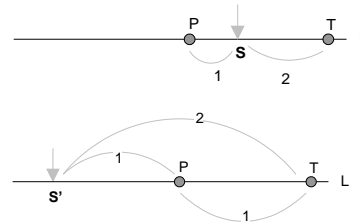
$= \frac{1}{4}(184+178+191+167)$
$= 180$
Thus, $183-180 = 3$ (lb) above the average.

47. $12×5 = 60$ (total of 5)
$2×9 = 18$ (total of 2)
$60 - 18 = 42$ (total of 3)
$42÷3 = \boxed{14}$

48. $\frac{3}{4}÷\frac{1}{3} = \boxed{2\frac{1}{4}}$

49. A

50. C
The only one less than $\frac{1}{2}$ is $\frac{18}{37}$.

51. $1.50

52. C
There are two desired points S and S'. As the figure below shows, S is an ideal point since PS:ST=1:2. S' is another ideal point since PS':S'T=1:2.

53. Next year is not a leap year.
$365÷7 = 52R1$
Saturday + 1 = $\underline{\text{Sunday}}$

54. $91 = □×(1+30\%)$
$□ = 91÷1.3 = \boxed{\$70.00}$

55. $174 ÷ 145 = 1.2$
$1.2 - 1 = 0.2 = 20\%$

56. $15\%×\frac{1}{3} = 5\%$
$5\%×500 = 25$
$500 + 25 = \$525$

57. $0.7×□ = 21.63$
$□ = 21.63÷0.7 = \boxed{\$30.90}$

58. total distance = $2×50 = 100$
total time = $1 + 1.5 = 2.5$
$100÷2.5 = \boxed{40 \text{ mph}}$

59. 1 square yard = $(3 \text{ ft})^2 = 9$ square ft
€135÷0.75 = \$180
$180÷9 = \boxed{\$20 \text{ per square foot}}$

60. B

61. D
P(at least one defective)
$= 1 - P$(all three perfect)
P(all three perfect) $= 0.8^3 = 0.512$
$1 - 0.512 = 0.488$

62. $0.1234 \times 1000 = 123.4$
$2n - 1 < 123.4 < 2n$
$n = \boxed{62}$

63. $2 \times 2 - 3 \times \frac{1}{2} = 2.5$

64. 16

65. E
$4x - 4 = 20$
$x = 6$
$2x = 12$

66. 9 and 5

67. $6 \div \frac{2}{5} = 15$

68. -2

69. The last occurrence of '*n*' is at
$\frac{1}{2}n(n+1)^{\text{th}}$ digit.
When $n = 9$, the last occurrence of $\boxed{9}$ is 45th digit.
For 2-digit numbers, starting from 10, each repetition of the number spins off 2 digits.
The first occurrence of 10 is at 46th digit.
The first occurrence of 11 is at $66^{\text{th}} = 46^{\text{th}} + 2 \times 10$ digit.
The first occurrence of 12 is at $88^{\text{st}} = 66^{\text{th}} + 2 \times 11$ digit.
The first occurrence of 13 is at $112^{\text{th}} = 88^{\text{st}} + 2 \times 12$ digit.
1 2 1 2 1 2 1 2 1 2 1 2 **1** 2
88 89 90 91 92 93 94 95 96 97 98 99 **100**
The answer is $\boxed{1}$.

70. Method I) (algebraic)
Let *s* be the coordinate of point S.
ST = 2SP
S is always on the left side of T, so $7 - s > 0$. (Why?)

$7 - s = 2|s - 1|$
$7 - s = \pm 2(s - 1)$
Case 1:
$7 - s = 2(s - 1)$
$3s = 9$
$s = \boxed{3}$

Case 1:
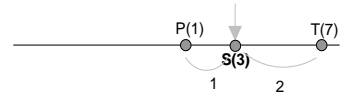$7 - s = -2(s - 1)$
$s = \boxed{-5}$

Method II) (geometric)
Case 1:

Case 2:

Drafted by www.MathEnglish.com

PSAT