AP Calc BC (Spring, 2024) Mock Exam 1 – Solution

1. [#17] Find the area inside the curve $r = 2\cos(3\theta)$ on the interval [#0, $\frac{\pi}{6}$].

2. [#21] Find $\frac{dy}{dx}$ if $y = \frac{\cos 2x - \sin^2 x}{2\sin^2 x}$.

3. [#26] Find the 3rd degree Maclaurin expansion for $f(x) = \ln(1 + 2x)$.

4. [#30] Find $\frac{dy}{dx}$ if $y = \frac{\sec x}{1-x^3}$.

   (A) $-\frac{\sec x \tan x}{3x^2}$

   (B) $\frac{(1-x^3)(\sec x \tan x)-(\sec x)(-3x^2)}{(1-x^3)^2}$

   (C) $\frac{(1-x^3)(\sec^2 x)-(\sec x)(-3x^2)}{(1-x^3)^2}$

   (D) $-\frac{\sec^2 x}{3x^2}$

5. [#31] Which of the following integrals gives the volume of the solid that results when the region between $y = 2x^2 + 4x - 9$ and $y = 3 - x$ is revolved around the line $y = -12$?

   (A) $\pi\int_{-\frac{3}{2}}^{4}\left[(2x^2+4x+3)^2-(15-x)^2\right]dx$

   (B) $\pi\int_{-4}^{\frac{3}{2}}\left[(2x^2+4x+3)^2-(15-x)^2\right]dx$

   (C) $\pi\int_{-\frac{3}{2}}^{4}\left[(15-x)^2-(2x^2+4x+3)^2\right]dx$

   (D) $\pi\int_{-4}^{\frac{3}{2}}\left[(15-x)^2-(2x^2+4x+3)^2\right]dx$

6. [#33] Find the value of c that is guaranteed by the Mean Value Theorem for $f(x) = x^3 + x - 8$ on the interval [#1, 2].

7. [#35] The rate of growth of fungus spores on a log can be modeled by $\frac{dy}{dt} = 0.85y$, where t is measured in days. If initially there are 515 fungus spores on the log, how many will there be after 5.8 days?

8. [#36] A rectangle is inscribed between the x-axis and $y=\sqrt{20-x^2}$. What is the maximum area of the rectangle?

9. [#37] Two particles are located
$x_1(t)=t^2+5t$
$x_2(t)=t^3$
At what time t are the two velocities equal on the interval [#0, 10]?

10. [#39] A box with a no top and rectangular sheet of tin with dimensions 8×14 in^2 by cutting identical squares from the four corners and folding up the sides. What is the maximum volume of the box?

11. [#40] Find the area of the region formed by the y-axis, y = sin x^2, and $y = e^{-x}$.

Answer Key

1. $\frac{1}{2}\int_0^{\frac{\pi}{6}} r^2 d\theta$
   $= \frac{1}{2}\int_0^{\frac{\pi}{6}} 4\cos^2 3\theta d\theta$
   $= 2\int_0^{\frac{\pi}{6}} \frac{\cos 6\theta+1}{2} d\theta$
   $= \int_0^{\frac{\pi}{6}} \cos 6\theta d\theta + \int_0^{\frac{\pi}{6}} d\theta$
   $= \frac{\pi}{6}$

2. Double angle formula:
   $\cos 2x = 1 - 2\sin^2 x = 2\cos^2 x - 1$

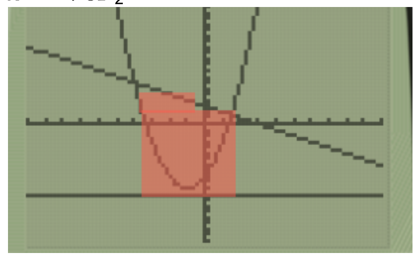
   $\frac{1 - 2\sin^2 x \ - \sin^2 x}{2\sin^2 x}$
   $= \frac{1 - 2\sin^2 x \ - \sin^2 x}{2\sin^2 x}$
   $= \frac{1 - 3\sin^2 x}{2\sin^2 x}$
   $= \frac{1}{2\sin^2 x} - \frac{3}{2}$

   $$\frac{dy}{dx} = -\frac{-2}{2}\frac{\cos x}{\sin^3 x} = -\frac{\cos x}{\sin x}\frac{1}{\sin^2 x} = -\cot x \cdot \csc^2 x$$

3. $\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + x^5 \ldots$
   $\frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 - x^5 \ldots$
   $\int \frac{1}{1+x} dx = \int 1 - x + x^2 - x^3 + x^4 - x^5 \ldots dx$
   $\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \ldots$
   $\ln(1+2x) = (2x) - \frac{1}{2}(2x)^2 + \frac{1}{3}(2x)^3 - \ldots$
   $= 2x - 2x^2 + \frac{8}{3}x^3$

4. B
   $y = \frac{1}{\cos x\,(1-x^3)}$
   Apply ln to both sides:
   $\ln y = -\ln \cos x - \ln(1-x^3)$
   $\frac{y'}{y} = \frac{\sin x}{\cos x} + \frac{3x^2}{1-x^3}$
   $y' = y(\tan x + \frac{3x^2}{1-x^3})$
   $y' = \frac{1}{\cos x\,(1-x^3)}\left(\tan x + \frac{3x^2}{1-x^3}\right) = \frac{(1-x^3)\sec x \tan x + 3x^2 \sec x}{(1-x^3)^2}$

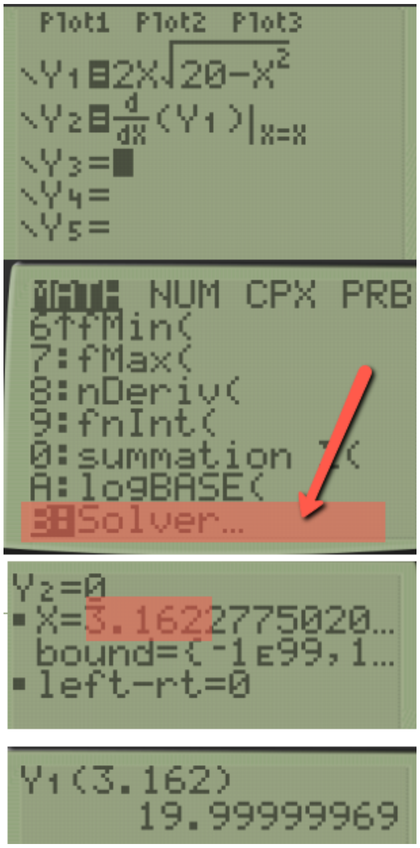
5. D
   $2x^2 + 4x - 9 = 3 - x$
   $2x^2 + 5x - 12 = 0$
   $(2x - 3)(x + 4) = 0$
   $x = -4$ or $\frac{3}{2}$



6. $f'(x) = 3x^2 + 1 = \frac{f(2)-f(1)}{2-1} = 8$
   $3x^2 = 7$
   $x = \sqrt{\frac{7}{3}}$

7. $\frac{dy}{y} = 0.85dt$
   $\ln y = 0.85t$
   $y = ce^{0.85t}$
   $y(0) = 515$
   $y(t) = 515e^{0.85t}$
   $y(5.8) =$

8.



9. $x_1'(t) = 2t + 5$

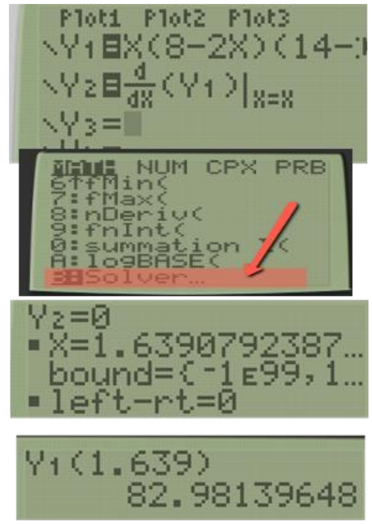
$x_2'(t) = 3t^2$

$3t^2 - 2t - 5 = 0$

$(3t - 5)(t + 1) = 0$

$t = \frac{5}{3}$

10. $V(x) = x(8 - 2x)(14 - 2x)$



11.