

AP Calc BC (Spring, 2024) Mock Exam 1 – Solution

1. [#17] Find the area inside the curve $r = 2\cos(3\theta)$ on the interval $[\#0, \frac{\pi}{6}]$.

2. [#21] Find $\frac{dy}{dx}$ if $y = \frac{\cos 2x - \sin^2 x}{2 \sin^2 x}$.
 - (A) $-\frac{\sec x \tan x}{3x^2}$
 - (B) $\frac{(1-x^3)(\sec x \tan x) - (\sec x)(-3x^2)}{(1-x^3)^2}$
 - (C) $\frac{(1-x^3)(\sec^2 x) - (\sec x)(-3x^2)}{(1-x^3)^2}$
 - (D) $-\frac{\sec^2 x}{3x^2}$

3. [#26] Find the 3rd degree Maclaurin expansion for $f(x) = \ln(1 + 2x)$.

4. [#30] Find $\frac{dy}{dx}$ if $y = \frac{\sec x}{1-x^3}$.
 - (A) $-\frac{\sec x \tan x}{3x^2}$
 - (B) $\frac{(1-x^3)(\sec x \tan x) - (\sec x)(-3x^2)}{(1-x^3)^2}$
 - (C) $\frac{(1-x^3)(\sec^2 x) - (\sec x)(-3x^2)}{(1-x^3)^2}$
 - (D) $-\frac{\sec^2 x}{3x^2}$

5. [#31] Which of the following integrals gives the volume of the solid that results when the region between $y = 2x^2 + 4x - 9$ and $y = 3 - x$ is revolved around the line $y = -12$?
 - (A) $\pi \int_{-\frac{3}{2}}^{\frac{3}{2}} [(2x^2 + 4x + 3)^2 - (15-x)^2] dx$
 - (B) $\pi \int_{-\frac{3}{2}}^{\frac{3}{2}} [(2x^2 + 4x + 3)^2 - (15-x)^2] dx$
 - (C) $\pi \int_{-\frac{3}{2}}^{\frac{3}{2}} [(15-x)^2 - (2x^2 + 4x + 3)^2] dx$
 - (D) $\pi \int_{-\frac{3}{2}}^{\frac{3}{2}} [(15-x)^2 - (2x^2 + 4x + 3)^2] dx$

6. [#33] Find the value of c that is guaranteed by the Mean Value Theorem for $f(x) = x^3 + x - 8$ on the interval $[\#1, 2]$.

7. [#35] The rate of growth of fungus spores on a log can be modeled by $\frac{dy}{dt} = 0.85y$, where t is measured in days. If initially there are 515 fungus spores on the log, how many will there be after 5.8 days?
 - (A) 515
 - (B) 515e^{4.93}
 - (C) 515e^{4.93}
 - (D) 515e^{4.93}

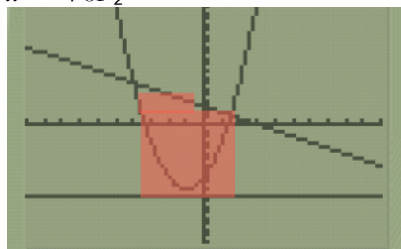
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8. [#36] A rectangle is inscribed between the x -axis and $y = \sqrt{20-x^2}$. What is the maximum area of the rectangle?
9. [#37] Two particles are located
 $x_1(t) = t^2 + 5t$
 $x_2(t) = t^3$
At what time t are the two velocities equal on the interval $[0, 10]$?
10. [#39] A box with a no top and rectangular sheet of tin with dimensions 8×14 in² by cutting identical squares from the four corners and folding up the sides. What is the maximum volume of the box?
11. [#40] Find the area of the region formed by the y -axis, $y = \sin x^2$, and $y = e^{-x}$.

Answer Key

$$\begin{aligned}
 1. \quad & \frac{1}{2} \int_0^{\frac{\pi}{6}} r^2 d\theta \\
 &= \frac{1}{2} \int_0^{\frac{\pi}{6}} 4 \cos^2 3\theta d\theta \\
 &= 2 \int_0^{\frac{\pi}{6}} \frac{\cos 6\theta + 1}{2} d\theta \\
 &= \int_0^{\frac{\pi}{6}} \cos 6\theta d\theta + \int_0^{\frac{\pi}{6}} d\theta \\
 &= \frac{\pi}{6}
 \end{aligned}$$

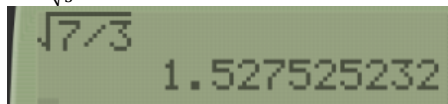
$$x = -4 \text{ or } \frac{3}{2}$$



2. Double angle formula:
 $\cos 2x = 1 - 2 \sin^2 x = 2 \cos^2 x - 1$

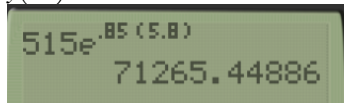
$$\begin{aligned}
 & \frac{1 - 2 \sin^2 x - \sin^2 x}{2 \sin^2 x} \\
 &= \frac{1 - 3 \sin^2 x}{2 \sin^2 x} \\
 &= \frac{1 - 3 \sin^2 x}{2 \sin^2 x} - \frac{3}{2} \\
 &= \frac{1}{2 \sin^2 x} - \frac{3}{2} \\
 & \frac{dy}{dx} = -\frac{2 \cos x}{2 \sin^3 x} = -\frac{\cos x}{\sin x \sin^2 x} \\
 & \quad \quad \quad = -\cot x \cdot \csc^2 x
 \end{aligned}$$

$$\begin{aligned}
 6. \quad & f'(x) = 3x^2 + 1 = \frac{f(2) - f(1)}{2 - 1} = 8 \\
 & 3x^2 = 7 \\
 & x = \sqrt{\frac{7}{3}}
 \end{aligned}$$



$$\begin{aligned}
 3. \quad & \frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + x^5 \dots \\
 & \frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 - x^5 \dots \\
 & \int \frac{1}{1+x} dx = \int 1 - x + x^2 - x^3 + x^4 - x^5 \dots dx \\
 & \ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \dots \\
 & \ln(1+2x) = (2x) - \frac{1}{2}(2x)^2 + \frac{1}{3}(2x)^3 - \dots \\
 & \quad \quad \quad = 2x - 2x^2 + \frac{8}{3}x^3
 \end{aligned}$$

$$\begin{aligned}
 7. \quad & \frac{dy}{y} = 0.85 dt \\
 & \ln y = 0.85t \\
 & y = ce^{0.85t} \\
 & y(0) = 515 \\
 & y(t) = 515e^{0.85t} \\
 & y(5.8) =
 \end{aligned}$$



4. B

$$\begin{aligned}
 & y = \frac{1}{\cos x (1-x^3)} \\
 & \text{Apply ln to both sides:} \\
 & \ln y = -\ln \cos x - \ln(1-x^3) \\
 & \frac{y'}{y} = \frac{\sin x}{\cos x} + \frac{3x^2}{1-x^3} \\
 & y' = y \left(\tan x + \frac{3x^2}{1-x^3} \right) \\
 & y' = \frac{1}{\cos x (1-x^3)} \left(\tan x + \frac{3x^2}{1-x^3} \right) = \frac{(1-x^3) \sec x \tan x + 3x^2 \sec x}{(1-x^3)^2}
 \end{aligned}$$

5. D

$$\begin{aligned}
 & 2x^2 + 4x - 9 = 3 - x \\
 & 2x^2 + 5x - 12 = 0 \\
 & (2x - 3)(x + 4) = 0
 \end{aligned}$$

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8.

Plot1 Plot2 Plot3
 $Y_1 = 2x\sqrt{20-x^2}$
 $Y_2 = \frac{d}{dx}(Y_1)|_{x=x}$
 $Y_3 =$
 $Y_4 =$
 $Y_5 =$

NUM CPX PRB
 6: fMin(
 7: fMax(
 8: nDeriv(
 9: fnInt(
 0: summation(
 A: logBASE(
Solver...

$Y_2 = 0$
 $X = 3.1622775020\dots$
 bound = $(-1E99, 1\dots)$
 left-rt = 0

$Y_1(3.162)$
 19.99999969

9.

$$x_1'(t) = 2t + 5$$

$$x_2'(t) = 3t^2$$

$$3t^2 - 2t - 5 = 0$$

$$(3t - 5)(t + 1) = 0$$

$$t = \frac{5}{3}$$

10. $V(x) = x(8 - 2x)(14 - 2x)$

Plot1 Plot2 Plot3
 $Y_1 = x(8-2x)(14-2x)$
 $Y_2 = \frac{d}{dx}(Y_1)|_{x=x}$
 $Y_3 =$
 $Y_4 =$
 $Y_5 =$

NUM CPX PRB
 6: fMin(
 7: fMax(
 8: nDeriv(
 9: fnInt(
 0: summation(
 A: logBASE(
Solver...

$Y_2 = 0$
 $X = 1.6390792387\dots$
 bound = $(-1E99, 1\dots)$
 left-rt = 0

$Y_1(1.639)$
 82.98139648

11.

$Y_1 = \sin(x^2)$
 $Y_2 = e^{-x}$
 $Y_3 =$

$Y_1 = \sin(x^2)$

$x = 0.73404255$ $y = 0.51312222$

$\int_0^{0.734} (Y_2 - Y_1) dx$
 0.3909069451