

Answer Key

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|---------------------|------------------------------|--|--|
| 1. 7.2 | 6. 9.89 | 11. {1, 2, 3, 4, 6, 8, 12, 16, 24, 48} | 16. {1, 2, 3, 6, 9, 18, 27, 54} |
| 2. 15.15 | 7. 14.12 | 12. {1, 7, 49} | 17. {1, 5, 11, 55} |
| 3. 9.36 | 8. 13.22 | 13. {1, 2, 5, 10, 25, 50} | 18. {1, 3, 19, 57} |
| 4. 11.64 | 9. 2.09 | 14. {1, 3, 17, 51} | 19. {1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60} |
| 5. 7.45 | 10. 0.18 | 15. {1, 2, 4, 13, 26, 52} | 20. {11, 13, 17, 19, 23, 29} |
| 21. $\frac{1}{2}$ | 26. $\frac{2}{3}$ | 31. $\frac{1}{7}$ | 36. $\frac{8}{11}$ |
| 22. $\frac{2}{9}$ | 27. $\frac{3}{10}$ | 32. $\frac{1}{8}$ | 37. $\frac{7}{11}$ |
| 23. $\frac{3}{7}$ | 28. $\frac{2}{5}$ | 33. $\frac{1}{3}$ | 38. $\frac{9}{11}$ |
| 24. $\frac{3}{4}$ | 29. $\frac{1}{5}$ | 34. $\frac{8}{9}$ | 39. $\frac{1}{4}$ |
| 25. $\frac{2}{7}$ | 30. $\frac{1}{9}$ | 35. $\frac{1}{6}$ | 40. $\frac{3}{5}$ |
| 41. $10\frac{2}{4}$ | 46. $4\frac{3}{4}$ | 51. 66 | 56. 12.3 |
| 42. $10\frac{4}{6}$ | 47. $1\frac{4}{6}$ | 52. 28 | 57. 80 |
| 43. $10\frac{6}{8}$ | 48. $2\frac{5}{8}$ | 53. 36 | 58. 360 |
| 44. $1\frac{8}{9}$ | 49. $2\frac{6}{9}$ | 54. 2 & 6 (R) | 59. 30.8 |
| 45. $3\frac{2}{6}$ | 50. $6\frac{5}{6}$ | 55. 6 | 60. 20 |
| 61. 2 | 66. 12.8 | 71. 9 tickets | 76. 5 (pounds) |
| 62. 7 | 67. 384 | 72. 30 | 77. \$1920 |
| 63. 216 | 68. 2 | 73. 20 quarts (smaller) | 78. \$10 |
| 64. 100 | 69. 12.8 | 74. 2 hrs | 79. 10 (girls) 16 (boys) |
| 65. 12 | 70. 20 | 75. 8 (pounds) | 80. 60 miles per hour |
| 81. = 46 inches | 86. $32\frac{1}{8}$ (pounds) | 91. 5 (greens) | 96. \$14 |
| 82. 5 hrs | 87. \$61 | 92. \$13 | 97. 21 (quarters) |
| 83. \$3.10 | 88. 33 (points) | 93. \$.50 | 98. \$10 |
| 84. 16 gallons | 89. \$30 | 94. 18 | 99. \$12 |
| 85. 16 races | 90. 5 (M) 10 (J) | 95. 8.4 miles | 100. \$41.45 |

MAP 239+ (T3) Issue 11

$101. 80 \div 4 = 20$

$20 \div 4 = \underline{5}$

$102. 5 \times 5 - 3 \times 2 = 19$

$19 \times 3 = \$57$

$103. 2.8 \times 15 = 1.4 \times 30 =$

14×3

$= \underline{\$42}$

$104. 132 \div 6 = 22$

$105. 91 \div 7 = 13$

$106. 90 \div 6 \times 7 = 105$

$107. 128 \div 4 = 32$

$108. 36 \div 4 = 9 \text{ in}$

109. Each quarter has 5 nickels.

$25 \times 5 = 125$

$110. 7 + 6 = 13$

$19 - 13 = 6 \text{ (yellow)}$

Answer Key

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|----------------------------------|---------------------|--------------------------|-------------------------------|
| 1. $6\frac{3}{2} = 7\frac{1}{2}$ | 6. $17\frac{1}{3}$ | 11. 2.4 | 16. 63 |
| 2. $13\frac{1}{2}$ | 7. $30\frac{2}{3}$ | 12. 27 | 17. 12 |
| 3. $27\frac{1}{2}$ | 8. $29\frac{2}{3}$ | 13. 77 | 18. 21 |
| 4. $50\frac{2}{3}$ | 9. $37\frac{1}{2}$ | 14. 117 | 19. 54 |
| 5. $16\frac{1}{2}$ | 10. $22\frac{1}{5}$ | 15. 27 | 20. 70 |
| 21. 4 | 26. 81 | 31. $\frac{1}{12}$ | 36. $\frac{3}{10}$ |
| 22. 33 | 27. 13 | 32. $\frac{3}{8}$ | 37. $\frac{5}{6}$ |
| 23. 24 | 28. 24 | 33. $\frac{3}{8}$ | 38. $\frac{9}{20}$ |
| 24. 21 | 29. 15 | 34. $\frac{11}{30}$ | 39. $\frac{5}{12}$ |
| 25. 63 | 30. 75 | 35. $\frac{14}{45}$ | 40. $\frac{7}{72}$ |
| 41. 2.5 | 46. 4.5 | 51. 60 | 56. 40 |
| 42. 300 | 47. 280 | 52. 27 | 57. 5 |
| 43. 260 | 48. 2.6 | 53. 0.001 | 58. 70 |
| 44. 0.7 | 49. 300 | 54. .001 | 59. 80 |
| 45. 36 | 50. 6 | 55. 120 | 60. 38 |
| 61. 6 | 66. 180 | 71. 24 (Quin) 8 (Rachel) | 76. 84 |
| 62. 30 | 67. 4 | 72. \$2 | 77. 7 |
| 63. 30 | 68. 10 | 73. 1760 feet | 78. 112 |
| 64. 30 | 69. 12 | 74. \$2.50 | 79. 5 |
| 65. 30 | 70. 180 | 75. \$580 | 80. 6 |
| 81. 3:55 pm | 86. N/A | 91. 50 | 96. A |
| 82. 8 hr & 50 min | 87. P, B, A, T | 92. 18 | 97. C |
| 83. <u>\$14.40</u> | 88. N/A | 93. 5 | 98. <u>\$10.00</u> |
| 84. \$3.10 | 89. 5 & 32 | 94. 65 | 99. <u>9 cups</u> |
| 85. 24 | 90. 1000 | 95. $\square = 5$ | 100. <u>21</u> |
| 101. 140 yd | 106. <u>75</u> | 111. 10:20 am | 116. 54 min |
| 102. 75° | 107. <u>14</u> | 112. 7:45 am | 117. 106 hours |
| 103. <u>4 2/3 dozen</u> | 108. B | 113. \$6.50 | 118. 15 hr |
| 104. <u>6 weeks</u> | 109. 6 hr | 114. 3:45 pm | 119. 1 hour 59 min |
| 105. \$1.50 | 110. 7 hr 10 min | 115. 42 min & 16 sec | 120. $12 - 3 \times 2 = 6$ am |

Answer Key

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|---|---|---|---|
| 1. 144 | 6. 3, 4, 5 | 11. $120^2 + 241 \times 1 =$
$14400 + 241 = 14641$ | 16. $15 + \frac{1}{2} = 15\frac{1}{2}$ |
| 2. 169 | 7. 5, 12, 13 | 12. $6\frac{2}{3}$ | 17. $40 + \frac{4}{3} = 41\frac{1}{3}$ |
| 3. 289 | 8. 8, 15, 17 | 13. $10\frac{1}{2}$ | 18. $80 + \frac{12}{5} = 82\frac{2}{5}$ |
| 4. 225 | 9. 7, 24, 25 | 14. $4 + \frac{1}{7} = 4\frac{1}{7}$ | 19. $60 + \frac{8}{3} = 62\frac{2}{3}$ |
| 5. 576 | 10. 9, 40, 41 | 15. $9 + \frac{1}{7} = 9\frac{1}{7}$ | 20. $6 + \frac{27}{5} = 11\frac{1}{5}$ |
| 21. $32\frac{1}{7}$ | 26. $47\frac{1}{7}$ | 31. $\frac{1}{24}$ | 36. $\frac{37}{105}$ |
| 22. $74\frac{2}{35}$ | 27. $75\frac{2}{35}$ | 32. $\frac{11}{160}$ | 37. $\frac{43}{160}$ |
| 23. $31\frac{1}{14}$ | 28. $32\frac{4}{9}$ | 33. $\frac{149}{360}$ | 38. $\frac{5}{18}$ |
| 24. $66\frac{2}{15}$ | 29. $76\frac{7}{18}$ | 34. $\frac{23}{140}$ | 39. $\frac{67}{360}$ |
| 25. $31\frac{1}{5}$ | 30. $60\frac{5}{32}$ | 35. $\frac{29}{70}$ | 40. $\frac{83}{300}$ |
| 41. 45 | 46. 0.49 | 51. 1800 | 56. 0.32 |
| 42. 9 | 47. 0.45 | 52. 1.6 | 57. 0.8 |
| 43. 50 | 48. 0.0125 | 53. 2.8 | 58. 0.12 |
| 44. 0.2 | 49. 0.225 | 54. 900 | 59. 0.0625 |
| 45. 0.35 | 50. 0.6 | 55. 0.012 | 60. 0.2 |
| 61. 240,000 | 66. 240 | 71. 80,000 | 76. 490,000 |
| 62. 21,000 | 67. 0.012 | 72. 800,000 | 77. 8,000 |
| 63. 0.032 | 68. 10 | 73. 0.15 | 78. 5 |
| 64. 0.28 | 69. 0.0056 | 74. 0.7 | 79. 0.0008 |
| 65. 1.6 | 70. 0.35 | 75. 150,000 | 80. 0.01 |
| 81. 4 in | 86. 24 in. | 91. 28 (in) | 96. $\boxed{35 \text{ (cm}^2\text{)}}$ |
| 82. 100 in. | 87. 250. | 92. $\boxed{33 \text{ (cm}^2\text{)}}$ | 97. 20 |
| 83. 200 in | 88. 13 (cm) | 93. $\boxed{33}$ | 98. 120 |
| 84. 628 in | 89. 60. | 94. $\boxed{35 \text{ (cm}^2\text{)}}$ | 99. 45 |
| 85. 27 in ² | 90. $\boxed{225 \text{ (total area)}}$ | 95. $\boxed{50 \text{ (cm}^2\text{)}}$ | 100. 160 |
| 101. $\frac{1}{2}(16 + 12) \times 6 = 84$ | 104. 6 miles = 3(2 miles)
$3(40 \text{ min}) = 120 \text{ min}$
$= 2 \text{ hrs}$ | 107. $43 - 1 = 42$
$42 \div 3 = 14$ | |
| 102. $0.01 \times \sqrt{100} =$
$0.01 \times 10 = 0.1$ | 105. $4 \times 1.5 = 6 \text{ mi}$
$4 \times (\frac{1}{2} \text{ hour}) = 2 \text{ hrs}$ | 108. $15 \times 8 = 120$ (panels a
day for a washer)
$120 \times 5 = 600$ panels
for 5 washers
$1200 \div 600 = 2 \text{ days}$ | |
| 103. $66 \times 15\% = 9.9$
$66 + 9.9 = \$75.90$ | 106. $3 \times 7 + 1 = 22$ | | |

MAP 259+ (T3) Issue 11

109. (a) false
No. Only a dog can do it.

(b)

- 1) a dog sails the boat.
- 2) the dog takes dog 2.
- 3) the dog takes a cat.
- 4) the dog takes cat 2.
- 5) the dog takes dog

3.

6) the dog takes cat 3.

110. 5:00 P.M.

The least common multiple of 30 and 40 is 120. So, you will hear both bell ring in 120 min, or at 5:00 P.M.

$$111. 900 \div 50000 = 1.8\%$$

$$80000 \times 1.8\% =$$

$$\$1440.00$$

$$112. \frac{500000}{100} \times 0.36 \times 5$$

$$= 5000 \times 0.36 \times 5$$

$$\text{(cancellation)}$$
$$= 50 \times 36 \times 5 \text{ (moving decimal)}$$

$$= 10 \times 180 \times 5$$

$$= 1800 \times 5$$

$$= \$9000.00$$

$$113. 1 + 20\% = 1.2$$

$$60 \times 1.2 = \$72$$

114. B

$$115. \frac{\text{tax}}{\text{cost without tax}} = \frac{54-50}{50} =$$

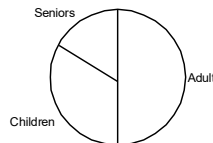
$$\frac{4}{50} = 8\%$$

Answer Key

1. $-6, 3x + 6$
2. $-2, 4x + 1$
3. $-9, 2x + 3$
4. $-16, 3x + 4$
5. $20, x - 4$
6. $5, 2x + 5$
7. $-4, 3x - 2$
8. $9, x + 3$
9. $16, 3x + 4$
10. $25, 2x + 5$
11. $7x^4 - 10x^3 - 15x^2$
12. $12x^3 + 12x^2 + 8x + 12$
13. $5x^4 - 3x^3 - 7x^2$
14. $6x^3 - 2x^2 + x - 2$
15. $18x^2 - 9x + 3$
16. $15x^2 - 14x + 8$
17. $8x^2 + 2x - 6$
18. $-4x^3 - 2x^2 - 12x$
19. $4x^3 + 16x^2 + 10x - 4$
20. $14x^3 + 14x^2 + 10x$
21. $32x^3 - 4x$
22. $4x^4 + 6x^2 - 2x$
23. $4x^4 - 6x^3 + 12x^2$
24. $18x^3 + 14x^2 + 10x$
25. $6x^4 - 9x^3 - 12x^2 - 3x$
26. $2x^3 + 2x^2 - 4x + 6$
27. $4x^4 - 4x^3 - 2x^2 - 6x$
28. $-36x^2 + 3x + 13$
29. $-2x^4 - 6x^3 - 4x^2 + 4x + 4$
30. $9x^3 + 10x^2 - 4x - 6$
31. $x^2 - 7x + 6$
32. $x^2 - x - 30$
33. $x^2 - 9x + 18$
34. $1.2x^2 - 4.8x - 14.4$
35. $15x^2 - 150x + 360$
36. $2.5x^2 - 7.5x - 45$

37. $1.5x^2 + 7x + 4$
38. $-2x^2 + \frac{1}{3}x + 4$
39. $12x^2 + 10x + 2$
40. $-3x^2 - 4.4x + 1$
41. $33x + 14$
42. $21x + 26$
43. 10
44. $4x^6$
45. $12x^2$
46. 6
47. $-11x$
48. $-7x^3$
49. $8x^2 - 3x + 18$
50. $-2x^2 + 3x - 4$
51. $5x^2 + 10x + 2$
52. $6x + 3 - 3x^2 + 12x + 6$
 $= -3x^2 + 18x + 9$
53. $2(3x^2 - 4x + 7) + 3(5x^2 + x + 11)$
 $= 6x^2 - 8x + 14 + 15x^2 + 3x + 33$
 $= 21x^2 - 5x + 47$
54. $3(2x^2 + 3) + 2(-4x^2 + 3x - 7)$
 $= 6x^2 + 9 - 8x^2 + 6x - 14$
 $= -2x^2 + 6x - 5$
55. $\frac{1}{2}(4x^3 + 8) + \frac{1}{3}(-6x + 3) - \frac{1}{5}(2x^3 - 20x^2 + 15)$
 $= 2x^3 + 4 - 2x + 1 - 0.4x^3 + 4x^2 - 3$
 $= 1.6x^3 + 4x^2 - 2x + 2$
56. $1.1(4x^3 + 7x + 8) - 3.2(4x^3 - 2x^2 + 3)$
 $= 4.4x^3 + 7.7x + 8.8 - 12.8x^3 + 6.4x^2 - 9.6$
 $= -8.4x^3 + 6.4x^2 + 7.7x - 0.8$
57. $1 - \frac{3}{5} = \frac{2}{5} = 2/5$

58. $35 \times \frac{2}{5} = 14$
59. $120 \div 6 = 20$
 $480 \div 20 = 24$ gal
60. 15%
61. See the chart below.



62. B
63. $3 \frac{3}{8}$
64. Method I)
 $\frac{80+70+60+90+80}{5} = 76$
- Method II)
Move 10 points from the last test to the third one. Now all marks exceed 70 points. The first one has a surplus of $80 - 70 = 10$ points, while the fourth one has a $90 - 70 = 20$ points. Therefore, the total surplus is $10 + 20 = 30$
 $30 \div 5 = 6$
 $70 + 6 = 76$
65. $39 \div 3 = 13$
 $13 + 9 = 22$
66. $9 \times 8 = 72$ in = 6 ft
67. A
SUV is more efficient.
SUV: $240 \div 8 = 30$ (mpg)
MIN: $299 \div 11 < 30$ (mpg)

68. $184 \div 8 = 23$ miles per gallon
69. $\frac{35}{40} \times 376 = \frac{7}{8} \times 376 = 47 \times 7 = \329
70. $8/7$
71. $\frac{2}{3} = 2/3$
72. $35 \div (1 - 30\%) = 35 \div 0.7 = \50
73. $0.8 \times 3000 = 2400$
or
 $2400 \div 0.8 = \$3,000$
74. $\frac{5000 - 4000}{5000} = 0.2 = 20\%$
75. $80 \times 30\% = 80 \times 0.3 = \24
76. $1 - 80\% = 20\%$
77. $60 \times 0.3 = \$18.00$
78. Method (I)
 $60 - 18 = \$42$
Method (II)
 $60 \times 0.7 = 42$
79. $6 \div 15\% = 6 \div 0.15 = 600 \div 15 = 40$ ft
80. $31.5 - 30 = \$1.50$
81. $1.5 \div 30 = 0.05 = 5\%$
82. $1 - 20\% = 1 - 0.2 = 0.8$
 $200 \times 0.8 = 160$
83. $2 \times (1 + 25\%) = 2 \times 1.25 = 2.5$ gal
 $= 2$ gal 4 pints
84. $200 \times (1 + 20\%) = 200 \times 1.2 = \240
85. $200 \times (1 + 15\%) = 200 \times 1.15 = 2 \times 115 = 230$
86. $240 \times (1 - 12\% - 18\%) = 168$

Answer Key

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|---|--|------------------------------|----------------------|
| 1. $a = -5, b = 1, c = -1$ | 19. $75 + \frac{5}{2} = 77\frac{1}{2}$ | 34. $y^2 - 4y - 21$ | 52. D |
| 2. $a = 7, b = 3, c = 14$ | 20. $48 + \frac{8}{3} = 50\frac{2}{3}$ | $y^2 + 4y - 21$ | 53. C |
| 3. $a = 7, b = 3, c = 33$ | 21. 6, 5, -6 | 35. $15x^2 + 41x + 14$ | 54. C |
| 4. $a = -1, b = 3, c = -27$ | 22. 4, 15, 9 | $15x^2 - 41x + 14$ | 55. A |
| 5. $a = 1, b = -3, c = 1$ | 23. 4, 2, 0, -1 | 36. $15x^2 + 29x - 14$ | 56. C |
| 6. $a = -5, b = 4, c = 1$ | 24. 6, 4, 1, -1 | $15x^2 - 29x - 14$ | 57. B |
| 7. $a = 7, b = -7, c = 3$ | 25. 3, 4, -7 | 37. $24x^2 + 41x - 35$ | vertex and |
| 8. $a = 1, b = -2, c = 6$ | 26. 3, 2, 11 | $24x^2 - 41x - 35$ | corresponding angles |
| 9. $a = 7, b = 4, c = 9$ | 27. 4, 1, 0 | 38. $3x^3 - 8x^2 - 22x + 24$ | 58. C |
| 10. $a = -1, b = -8, c = 9$ | 28. 3, 3, -1 | 39. $x^3 + 8x^2 + 14x + 4$ | 59. C |
| 11. $\frac{5}{3} = 1\frac{2}{3}$ | 29. 1, -1, 1 | 40. $x^6 + 8x^4 + 14x^2 + 4$ | 60. D |
| 12. $\frac{21}{5} = 4\frac{1}{5}$ | 30. 2, 2, 1 | 41. $(x + 1)(x + 2)$ | 61. B |
| 13. $6 + \frac{8}{5} = 7\frac{3}{5}$ | 31. $(x - 2)(x + 3) = x^2 +$
$(3 - 2)x + (-2)(3) = x^2 +$
$x - 6$
$(x + 2)(x - 3) = x^2 - x$
$- 6$ | 42. $(x + 1)(x + 8)$ | 62. B |
| 14. $24 + \frac{8}{7} = 25\frac{1}{7}$ | 32. $(3x - 2)(2x + 3) = 6x^2$
$+ 5x - 6$
$(3x + 2)(2x - 3) = 6x^2$
$- 5x - 6$ | 43. $(x + 3)(x + 10)$ | 63. D |
| 15. $12 + \frac{6}{5} = 13\frac{1}{5}$ | 33. $n^2 - 7n + 12$
$n^2 + 7n + 12$ | 44. $(x - 8)(x - 4)$ | 64. B |
| 16. $18 + \frac{12}{7} = 19\frac{5}{7}$ | | 45. $(x - 2)(x + 9)$ | 65. B |
| 17. $30 + \frac{5}{3} = 31\frac{2}{3}$ | | 46. $(4x - 5)(3x + 1)$ | 66. A |
| 18. $32 + \frac{8}{5} = 33\frac{3}{5}$ | | 47. $(3x + 2)(x - 10)$ | 67. B |
| | | 48. $(2x - 9)(x - 3)$ | 68. -64 |
| | | 49. $(x + 8)(3x + 4)$ | 69. D |
| | | 50. $(2x + 3)(5x + 1)$ | 70. B |
| | | 51. B | |

Answer Key

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|--------------------|--|
| 1. $f(0) = 75$ | 23. 0.064 |
| $f(4) = 0.12$ | 24. 0.25 |
| $f(x) = 75(0.2^x)$ | 25. 0.125 |
| 2. $f(0) = 2$ | 26. 2 |
| $f(6) = 128$ | 27. 2 |
| $f(x) = 2(2^x)$ | 28. 2 |
| 3. $f(0) = -4$ | 29. 6 |
| $f(4) = -64$ | 30. 2 |
| $f(x) = -4(2^x)$ | 31. 4 |
| 4. $f(0) = 60$ | 32. 3 |
| $f(-1) = 100$ | 33. 2 |
| $f(x) = 60(0.6^x)$ | 34. 2 |
| 5. $f(0) = 0.12$ | 35. $1\frac{1}{2}$ |
| $f(4) = 75$ | 36. $\frac{5}{6} + \frac{2}{3} = 1\frac{1}{2}$ |
| $f(x) = 0.12(5^x)$ | 37. $\frac{7}{10} + \frac{1}{2} = 1\frac{1}{5}$ |
| 6. 2, 7, 3 | 38. $\frac{9}{3} + \frac{2}{3} = 3\frac{2}{3}$ |
| 7. 2, 1, -6 | 39. $\frac{7}{8} + \frac{1}{3} + \frac{1}{2} = 1\frac{17}{24}$ |
| 8. 2, -1, 1, 1 | 40. $\frac{6}{7} + \frac{2}{5} + \frac{2}{1} = 3\frac{9}{35}$ |
| 9. 2, -1, -8, -3 | 41. $3(3x - 8)(7x + 6)$ |
| 10. 1, -5, -2 | 42. $5(5x + 8)(6x + 5)$ |
| 11. 2, 5, 18 | 43. $4(x + 6)(8x - 1)$ |
| 12. 2, -8, 7 | 44. $2(3x + 4)(6x + 5)$ |
| 13. 8, -5, -6 | 45. $-5(5x + 2)(3x - 5)$ |
| 14. 2, 1, 1 | |
| 15. 2, 2, -1 | |
| 16. 1.96 | |
| 17. 0.04 | |
| 18. 0.008 | |
| 19. 0.0016 | |
| 20. 0.09 | |
| 21. 0.027 | |
| 22. 0.16 | |

Advanced Math (T3) Issue 11

46. 6 min

stamping machine	fraction of a part that can be completed in one sec
older model	$\frac{1}{18}$
newer model	$\frac{1}{12}$
both models	$\frac{1}{18} + \frac{1}{12} = \frac{5}{36}$

$$\frac{50}{\frac{1}{18} + \frac{1}{12}} = 50 / \frac{5}{36} = 360 \text{ sec} = 6 \text{ min}$$

Pump	part of the tank can be filled in one min
Pump A	$\frac{1}{30}$
Pump B	$\frac{1}{45}$
both Pumps	$\frac{1}{30} + \frac{1}{45} = \frac{1}{18}$

47. It takes 18 min for both pumps to fill the tank.

$$48. \frac{1}{30} + \frac{1}{45} - \frac{1}{90} = \frac{2}{45}$$

It takes 22.5 min for both pumps to fill the tank while the drain pipe is open.

$$49. \frac{70}{\frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{6}} = \text{span style="border: 1px solid black; padding: 2px;">150 sec}$$

50. Let x be the distance in km.

$$\begin{aligned} \frac{x}{15} &= \frac{x}{30} + \frac{1}{4} \\ 4x &= 2x + 15 \\ x &= \text{span style="border: 1px solid black; padding: 2px;">7.5 km} \end{aligned}$$

51. Let x km be the length of the first leg (part) of the race, thus the remainder is $12 - x$ km. We have

$$\begin{aligned} \frac{x}{9} + \frac{12-x}{10} &= 1\frac{1}{4} \\ 20x + 18(12-x) &= 225 \\ 2x &= 9 \\ x &= \text{span style="border: 1px solid black; padding: 2px;">4.5 km} \end{aligned}$$

52. Let x miles be the length of city driving, thus $150 - x$ miles be the length of the highway driving. The total number gallons of gas is

$$\frac{150}{20} = 7.5$$

He spend $\frac{x}{18}$ and $\frac{150-x}{24}$ gallons of gas on city and highway driving, respectively. Thus, we have

$$\frac{x}{18} + \frac{150-x}{24} = 7.5$$

$$4x + 3(150 - x) = 540$$

$$x = \text{span style="border: 1px solid black; padding: 2px;">90 mi}$$

53. Let x g of pure hydrogen peroxide be added. Thus, we have

$$x + 240 \times 0.45 = (x + 240) \times 0.7$$

$$x + 108 = 0.7x + 168$$

$$0.3x = 60$$

$$x = \text{span style="border: 1px solid black; padding: 2px;">200 g}$$

54. Let x (g) of nitric solution be added. Then, we have

$$80\% \times x + 12\% \times 175 = 60\% \times (x + 175)$$

$$0.8x + 21 = 0.6x + 105$$

$$0.2x = 84$$

$$x = \text{span style="border: 1px solid black; padding: 2px;">420 g}$$

$$55. \frac{1 - 6(\frac{2}{25})}{\frac{2}{25} + \frac{1}{20}} = 4$$

$$6 + 4 = \text{span style="border: 1px solid black; padding: 2px;">10 min}$$

56. Let x (min) be the length of time before the first machine breaks down. Therefore, we have

$$\frac{1 - \frac{x}{18}}{\frac{1}{60} + \frac{1}{60}} = 20$$

$$1 - \frac{x}{18} = \frac{2}{3}$$

$$18 - x = 12$$

$$x = \text{span style="border: 1px solid black; padding: 2px;">6 min}$$

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57. For the first 30 km, she spent $\frac{3}{4}$ hr.

Let x (km) be the distance she traveled further at the speed of 60 km/h. Then, the total time she traveled is

$$\frac{3}{4} + \frac{x}{60} = \frac{45+x}{60}$$

so the average speed is

$$\frac{\text{total distance}}{\text{total time}} = \frac{30+x}{\frac{3}{4} + \frac{x}{60}} = 54$$

$$30 + x = 54\left(\frac{3}{4} + \frac{x}{60}\right)$$

$$30 + x = 54 \times \frac{45+x}{60}$$

$$10(30 + x) = 9(45 + x)$$

$$300 + 10x = 405 + 9x$$

$$x = \boxed{105 \text{ km}}$$

58. Let x (min) be needed for the second copy machine. Every minute the total part of the work performed is

$$\frac{1}{30} + \frac{1}{x} = \frac{1}{18}$$

$$9x + 270 = 15x$$

$$6x = 270$$

$$x = \boxed{45 \text{ min}}$$

59. Let x (hr) be the length of time needed by the third machine alone.

Then, we have

$$\frac{2}{15} + \frac{1}{x} = \frac{1}{3}$$

$$2x + 15 = 5$$

$$x = \boxed{5 \text{ hr}}$$

60. Let x (hr) be the length of time needed to complete the job by a newer model machine, then $x + 6$ (hr) be needed by an older model machine. The fraction of work that can be accomplished by 2 newer model machines is $\frac{2}{x}$, while the fraction of work that can be accomplished by 3 older model machines is $\frac{3}{x+6}$. Since the combined work accomplished by 5 of these machines in one single hour is the whole job, we have

$$\frac{3}{x+6} + \frac{2}{x} = \frac{1}{1}$$

$$3x + 2(x + 6) = x(x + 6)$$

$$5x + 12 = x^2 + 6x$$

$$x^2 + x - 12 = 0$$

$$(x + 4)(x - 3) = 0$$

$$x = \boxed{3 \text{ hr}} \text{ (-4 not desired)}$$

61. Let x (mi/h) be the speed going to and $x - 6$ on the way returning from the meeting.

$$\frac{72}{x} + \frac{2}{15} = \frac{72}{x-6}$$

$$72(x - 6) + \frac{2}{15}x(x - 6) = 72x$$

$$\frac{2}{15}x(x - 6) - 432 = 0$$

$$x(x - 6) - 3240 = 0$$

$$x^2 - 6x - 3240 = 0$$

$$(x - 60)(x + 54) = 0$$

$$x = \boxed{60 \text{ km/h}} \text{ (-54 not desired)}$$

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62. Let x (hr) be the length of time he worked before raise. His hourly rate was $\frac{63}{x}$ ¢/h, while his current rate is $\frac{63}{x} + 75$ ¢/h. We have

$$\left(\frac{63}{x} + \frac{3}{4}\right)(x - 2) = 68$$

$$(3x + 252)(x - 2) = 272x$$

$$3x^2 - 26x - 504 = 0$$

$$(3x + 28)(x - 18) = 0$$

$$x = \boxed{18 \text{ hr}} \text{ (-28/3 not desired)}$$

63. Let x km/h be the speed of the current, then the combined speed of going up stream is $15 - x$ and the combined speed of going down stream is $15 + x$. Thus, the total time equals the sum of up stream time and down stream time:

$$T_{\text{total}} = T_{\text{up stream}} + T_{\text{down stream}}$$

$$\frac{5}{6} = \frac{6}{15-x} + \frac{6}{15+x}$$

$$\frac{5}{6} = \frac{180}{(15-x)(15+x)}$$

$$\frac{1}{6} = \frac{36}{(15-x)(15+x)}$$

$$225 - x^2 = 216$$

$$x^2 = 9$$

$$x = \boxed{3 \text{ km/h}} \text{ (-3 not desired)}$$

64. Let x km/h be the speed of the boat. Then, the combined up stream speed is $x - 2$ and the combined down stream is $x + 2$. Since

$$T_{\text{up stream}} + T_{\text{down stream}} = \text{paddle 25 km in still water}$$

$$\frac{12}{x-2} + \frac{12}{x+2} = \frac{25}{x}$$

$$24x^2 = 25(x^2 - 4)$$

$$x^2 = 100$$

$$x = \boxed{10 \text{ km/hr}} \text{ (-10 not desired)}$$

65. We have the following relations:

	# days needed	part of work accomplished a day
day laborer	$x + 3$	$\frac{1}{x+3}$
apprentice	x	$\frac{1}{x}$
master	$x - 3$	$\frac{1}{x-3}$

The part of work accomplished by master in 7 days is

$$\frac{7}{x-3}$$

The part of work accomplished by apprentice and day laborer in 6 days is

$$6\left(\frac{1}{x} + \frac{1}{x+3}\right)$$

Thus, we have

$$\frac{7}{x-3} = 6\left(\frac{1}{x} + \frac{1}{x+3}\right)$$

$$7x(x + 3) = 6(x - 3)(2x + 3)$$

$$7x^2 + 21x = 6(2x^2 - 3x - 9)$$

$$7x^2 + 21x = 12x^2 - 18x - 54$$

$$5x^2 - 39x - 54 = 0$$

$$(5x + 6)(x - 9) = 0$$

$$x = \boxed{9 \text{ days}} \text{ (-1.2 not desired)}$$

$$66. \frac{1}{\frac{1}{0.4} - \frac{1}{0.5} + \frac{1}{4.5} - \frac{1}{1.5}}$$

$$= \frac{36}{90 - 72 + 8 - 24}$$

$$= \boxed{18 \text{ hr}}$$

$$67. \frac{5}{2}$$

$$68. \frac{5}{3}$$

$$69. \frac{8}{9}$$

$$70. \frac{8}{5}$$

$$71. \frac{7}{4}$$

$$72. \frac{8}{7}$$

$$73. \frac{7}{6}$$

$$74. \frac{8}{7}$$

$$75. \frac{9}{7}$$

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76. $\frac{7}{6}$

77. $\alpha + \beta = -1$

78. $\alpha\beta = -1$

79. $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 3$

80. $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = -1 - 3 = -4$

81. $\alpha - \beta = ((\alpha + \beta)^2 - 4\alpha\beta)^{\frac{1}{2}} = \sqrt{5}$

82. $\alpha^3\beta + \alpha\beta^3 = \alpha\beta(\alpha^2 + \beta^2) = -1 \times 3 = -3.$

83. $\alpha + \beta = -\frac{3}{2}$

84. $\alpha\beta = \frac{5}{2}$

85. Let a and b be the length of the rectangle. Since we have $2(a + b) = p$ or

(i) $a + b = \frac{1}{2}p$, and

(ii) $ab = q$

From (i) and (ii), we have $a^2 + b^2 = (a + b)^2 - 2ab = \frac{1}{4}p^2 - 2q$, thus the length

of the diagonal is $\sqrt{\frac{1}{4}p^2 - 2q}$

86.
$$\begin{aligned} & \frac{(5-\sqrt{6x})(\sqrt{6x}+5)}{7x-2} + \frac{3x+2}{7x-2} \\ &= \frac{25-6x}{7x-2} + \frac{3x+2}{7x-2} \\ &= \frac{27-3x}{7x-2} \end{aligned}$$

87.
$$\begin{aligned} 8(x + 3a) &= 2(6 + 2a) \\ 2(x + 3a) &= 3 + a \\ 2x &= 3 - 5a \\ x &= \frac{3-5a}{2} \end{aligned}$$

88.
$$\begin{aligned} x^2 + bx + 9 + y^2 &= 16 \\ (x + 3)^2 + y^2 &= 4 \\ b &= \boxed{6} \end{aligned}$$

89.
$$\begin{aligned} (-3i - i)^2 - (-1 + 2i)^2 & \\ &= (4i)^2 - (-1 + 2i)^2 \\ &= -16 + 3 + 4i \\ &= -13 + 4i \end{aligned}$$

90.
$$\begin{aligned} -5.6 < x - 3 < 7 \\ -2.6 < x < 10 \\ -85.6 < 33x < 330 \end{aligned}$$