Answer Key

1. $x \cdot x^3 \cdot x^5 \cdot x^7 = x^{16}$
2. $-(-a^2)^2(a^3)^3 = -a^4a^9 = -a^{13}$
3. $(x^2)^2(x^3)^3 = x^{13}$
4. $3(-2xy^2z^3)^3 = -24x^3y^6z^9$
5. $3t^6$
6. $\frac{3}{-10x^3}$
7. $\frac{3x}{2y^4}$
8. $-\frac{1}{2xy^2}$
9. $\frac{15}{2x}$
10. $\frac{5t^3}{3s^2}$
11. $\frac{10b}{3}$
12. s^{14}
13. t^2
14. $\frac{1}{w^8}$
15. $\frac{9x^6}{y^8}\frac{x^{21}}{6^3y^{15}} = \frac{x^{27}}{24y^{23}}$
16. $\frac{1}{a^5}$
17. $(\frac{y}{2x})^7 = \frac{y^7}{2^7x^7}$
18. $\frac{1}{c^9}$
19. 1
20. $576\ x^6y^{12}z^{18}$
21. $(\frac{y}{2x})^{-7} = (\frac{2x}{y})^7$
22. $(\frac{c^{-6}}{(c^{15})})^{-1} = (\frac{1}{(c^{21})})^{-1} = c^{21}$
23. $-2x^{-2} = \frac{-2}{x^2}$
24. $6t^4/(3t^{-3}) = 2t^{4-(-3)} = 2t^7$
25. $(-4p^{-4})(3p^{-3})/(3p^{-2}4p^5) = \frac{-12p^{-7}}{12p^3} = \frac{-1}{p^{10}}$
26. $2\sqrt{11}$
27. $8\sqrt{2}$
28. $12\sqrt{2}$
29. $18\sqrt{2}$
30. $18\sqrt{3}$
31. 30
32. $\frac{y}{2x^2}\sqrt{\frac{y}{3x}}$
33. $10s^2t^2\sqrt{(15st)}$
34. $\frac{6}{7xy^3}\sqrt{\frac{2}{x}}$
35. $(30\sqrt{15})x^2y$
36. 8000
37. 8
38. 64
39. 10
40. 100
41. 40
42. 64000
43. 50
44. 125000
45. 20
46. Y = 6/7X - 3
47. Y = 4/7X - 13/7
48. Y = 1/3X + 8/3
49. -X - 6Y = 20
50. 5X + 6Y = 9
51. 20
52. 84.5
53. 136
54. 62.5
55. $1.5\sqrt{13}$
56. $x^2 + 6x + 9$
57. $x^2 - 6x + 9$
58. $4x^2 - 4x + 1$
59. $9x^2 - 6x + 1$
60. $25x^2 + 2x + 0.04$
61. $(x + 1)^2$
62. $(x + 2)^2$
63. $(x - 3)^2$
64. $(x - 9)^2$
65. $(x + 4)^2$
66. 16, 8, 17
67. 25, –10, 15
68. 4, –20, –10
69. 2, 16, 19
70. 25, –20, 28
71. 1, 3, 4, –3
72. 2, 1, 6, –62

73. 2, 3, –5, –74

74. 2, 3, 5, –30

75. 2, 4, –5, –33

76. 72

77. 250000+12000 + 144 = 262144

78. $4\sqrt{xy} = 12$

79. $4^{x+2} + 4^{x+5} = 65(4^{x+2}) = 130$
$4^{x+2} = 2$
$x + 2 = 0.5$
$x = -1.5$

80. 2

81. 385

82. C

83. 9

84. $(x + \frac{1}{x})^2 = x^2+2+\frac{1}{x^2} = 9$
$x + \frac{1}{x} = \sqrt{7}$
$x^2 - 1 + \frac{1}{x^2} = 6$
$x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)(x^2 - 1 + \frac{1}{x^2}) = 6\sqrt{7}$

85. Method I)
Let $a = 2019$
$2021 = a + 2$
$(a + 2)^3 = a^3 + 6a^2 + 12a + 8$
$2021^3 - a^3 = 6a^2 + 12a + 8$
$(6a^2 + 12a + 8 – 2)\div 6 = a^2 + 2a + 1 = (a + 1)^2$
$\sqrt{\frac{x-2}{6}} = a + 1 = 2020$

Method II)
Let $a = 2021$
$b = 2019$
$x = a^3 – b^3 = (a–b)(a^2 + ab + b^2) = (a-b)^3 + 3ab(a-b)$
$a – b = 2$
$x – 2 = 8 + 3ab(2) – 2 = 6ab + 6 = 6(ab + 1)$
$\frac{x-2}{6} = ab + 1$
$a = 2020 + 1$
$b = 2020 – 1$
$ab + 1 = 2020^2 – 1 + 1 = 2020^2$
$\sqrt{\frac{x-2}{6}} = 2020$

Answer Key

1. $2^4 \times 3^6$
2. $2^7 \times 5^3$
3. 2^9
4. 3
5. 2^{24}
6. $5^1 \times 2^3 \times 5^2 \times 2^4 = 2^7 \times 5^3$
7. $9^2 = 3^4$
8. $8^2 = 2^6$
9. -81
10. 81
11. 981
12. $(2.3 \times 0.5) \times 4 = 2.3 \times (0.5 \times 4) = 2.3 \times 2 = 4.6$
13. $x^{4+5-3} = x^6$
14. $x^{5+10-(4+2)} = x^{15-6} = x^9$
15. $-8t^5$
16. $-81t^{12} = 81t^6 \cdot -t^6$
17. $1/x^7$
18. $1/x^9$
19. $5/x^4$
20. $(3)(4)^2(2)^3$
    $= (3)(16)(8)$
    $= 384$
21. $4^6 \div 2^3 = 2^{12} \div 2^3 = 2^9$
22. 8^3 5/3
23. 2^6
24. $16^2 \times 2^3 = (2^4)^2 \times 2^3 = 2^8 \times 2^3 = 2^{11}$
25. 8^4
26. $4^4 = (2^2)^4 = 2^8$
27. $4^3 = 2^6$ 2
28. $4^2 \times 2^3 = 2^4 \times 2^3 = 2^7$
29. 4^6
30. 16^3
31. 8^3 5
32. 3^3
33. 27^6
34. 27^3
35. $3^6 \div 9^2 = 3^6 \div 3^4 = 3^2$
36. $25^3 = (5^2)^3 = 5^6$
37. $25^2 \times 5^4 = 5^4 \times 5^4 = 5^8$
38. 25^3
39. 5^7
40. 125^6
41. $(-2)^2 = 4$
42. $-2(-x^5)^3 = -2(-x^{15}) = 2x^{15}$
43. $(x^{-1})^2 = (\frac{1}{x})^2 = \frac{1}{x^2}$
44. $\frac{x^4}{x^6} = \frac{1}{x^2}$
45. 3^3
46. $\frac{8}{125}$
47. $-\frac{1}{32}$
48. $\frac{1}{16x^4}$
49. $\frac{1}{16x^4}$
50. $\frac{-1}{32x^5}$
51. $-32x^5$
52. -1
53. 1
54. 1/64
55. $\frac{1}{(-2x)^3} = \frac{1}{-8x^3}$
56. 1
57. $\frac{4}{9}$
58. $\frac{-1}{t^6}$
59. $\frac{1}{(-2x)^4} = \frac{1}{16x^4}$
60. $n = 2$
61. Using longhand multiplication, we have the following:

$$\begin{array}{r} A - B \\ \times\ A + B \\ \hline AB - B^2 \\ A^2 - AB \\ \hline A^2 \qquad - B^2 \end{array}$$

62. Try something different than longhand multiplication. Be fancy! Think of 98 = 100 - 2, what then 102 = ? Be smart: 102 = 100 + 2. That means we assume A = 100, B = 2.
    Now, $98 \times 102 = (100 - 2)(100 + 2)$
    $= 100^2 - 2^2$
    $= 10{,}000 - 4$
    $= 9{,}996$
63. $(x + 3)(x - 3) = x^2 - 9$

64. $(2x + 5)(2x - 5) = 4x^2 - 25$
65. $97\times103 = 10{,}000 - 9 = 9{,}991$
66. $197\times203 = 40{,}000 - 9 = 39{,}991$
67. $306\times294 = 90{,}000 - 36 = 89{,}964$
68. $208\times192 = 40{,}000 - 64 = 39{,}936$
69. $1234567890 = 1234567891 - 1$
    $1234567892 = 1234567891 + 1$
    Let $a = 1234567891$.
    $a^2 - (a - 1)(a + 1)$
    $= a^2 - (a^2 - 1)$
    $= \boxed{1}$
70. $9x^2 - 18x + 9$
71. $4x^2 - 8x + 4$
72. $25x^2 - 70x + 49$
73. $x^2 - 2x + 1$
74. $36x^2 - 60x + 25$
75. $25x^2 - 20x + 4$
76. $144x^2 - 120x + 25$
77. $x^2 + 5x\text{-}24$
78. $x^2 + 3x\text{-}54$
79. $x^2\text{-}6x\text{-}7$
80. $4n^2 + 27n + 18$
81. $x^2 - 1$
82. $4x^2 - 81$
83. $50x^2 - 18$
84. $\frac{1}{4}x^2 - 9$
85. $\frac{1}{3}x^2 - 3$
86. $x^2 - 25$
87. $x^2 - 100$
88. $x^2 - 121$
89. $x^2 - 4y^2$
90. $x^2 - 36y^2$
91. $9x^2 - y^2$
92. $49x^2 - 64y^2$
93. $36a^2 - 25b^2$
94. $x^2 - 1$
95. $x^2 - 4$
96. $x^2 - 9$
97. $2x^2 - 32$
98. $x^4 - 1$
99. $x^4 - 16$
100. $x^4 - y^4$

Answer Key

1. $\frac{1}{3^7}$
2. $\frac{1}{3^3}$
3. $\frac{9}{4}$
4. x
5. $\frac{-1}{3x}$
6. 0
7. $\frac{-27x}{4x^2}$
8. $(-4p)^2 \cdot (3p^6) = 48p^8$
9. $x^2x^5y^2y^3 = x^7y^5$
10. $\frac{1}{x^3}$
11. $\frac{3y^3}{x^3}$
12. $2x^3y$
13. $\frac{3y}{2x^{11}}$
14. $\frac{-8x^8}{3y^8}$
15. $(2x^4y)^{-1}(2x^2y^2)(5xy) = \frac{1}{2x^4y} \cdot 10x^3y^3 = \frac{5y^2}{x}$
16. $10^2 = 100$
17. $51^2 = 2601$
18. $(2\frac{2}{3} \times 3\frac{3}{4} \times 5\frac{1}{10})^2 = 51^2 = 2601$
19. Incorrect. It should be $(2\frac{2}{3})^2 = (\frac{8}{3})^2 = \frac{64}{9}$
20. $= x^2yz^5 + 3x^2yz^5 + 2xy^2z^5 - 7xy^2z^5$
    $= 4x^2yz^5 - 5xy^2z^5$
21. 216
22. -108
23. 3^7
24. 3^{14}
25. $(\frac{1}{3^3})^{-2} = 3^6$
26. $-x^6$
27. $-27y^3$
28. $16t^8$
29. $p^{-6}p^6 = p^0 = 1$
30. $x^{-1} \cdot x^2 \cdot x^{-3} \cdot x^4 \cdot x^{-5} = 1/x^3$
31. $1/x^{14}$
32. $15a^2x^4$
33. $-x^{14}y^{21}$
34. $\frac{24y^8}{x^5}$
35. $-189a^{10}b^4c^4$
36. $\frac{y^4}{x^2}$
37. $\frac{x^3}{3}$
38. $\frac{-5y^6}{24x^8}$
39. $\frac{9y^4}{4x^7}$
40. $\frac{-4y^3}{x^3}$
41. x^7
42. $-120x^{15}$
43. x^{15}
44. x^7
45. $32w^{10}$
46. $-x^{17}$
47. s^{24}
48. $120s^9$
49. $-c^6$
50. $30a^{-6}b^3 = \frac{30b^3}{a^6}$
51. $\frac{10s^2}{t^3}$
52. $-30xy^3$
53. $9x^6y^8$
54. $25a^4b^6$
55. $-8a^5x^4$
56. $-20x^7y^4$
57. $\frac{-p^{10}}{p^8} = -p^2$
58. $\frac{25a^4b^6}{-1000a^{12}b^6} = \frac{1}{-40a^8}$
59. $\frac{63a^6b^3c^4}{-3a^4b} = -21a^2b^2c^4$
60. $\frac{t^2 \cdot 3t^4}{4t^3} = \frac{3t^6}{4t^3} = \frac{3t^3}{4}$
61. $(6x + 3)^2$
62. $(4x + 5)^2$
63. $(3x + 2)^2$
64. $(7x + 5)^2$
65. $(7x - 0.3)^2$
66. $(9x + 4)^2$
67. $(x + 4)^2$
68. $(9x - 2)^2$
69. $(4x - 2)^2$
70. $(3x + 5)^2$

71. $(x - 5)^2$
72. $(6x - 1)^2$
73. $(5x + 1)^2$
74. $(7x - 2)^2$
75. $(7x + 0.6)^2$
76. $(6x + 1)^2$
77. $(6x + 2)^2$
78. $(7x + 5)^2$
79. $(8x + 5)^2$
80. $(x + 5)^2$

Answer Key

1. x^7
2. x^{15}
3. x^7
4. $32w^{10}$
5. s^{24}
6. $-c^6$
7. $-x^{17}$
8. $120s^9$
9. $-120x^{15}$
10. $30a^{-6}b^3 = \frac{30b^3}{a^6}$
11. $\frac{10s^2}{t^3}$
12. $-30xy^3$
13. $9x^6y^8$
14. $25a^4b^6$
15. $-8a^5x^4$
16. $-20x^7y^4$
17. $\frac{-p^{10}}{p^8} = -p^2$
18. $\frac{25a^4b^6}{-1000a^{12}b^6} = \frac{1}{-40a^8}$
19. $\frac{63a^6b^3c^4}{-3a^4b} = -21a^2b^2c^4$
20. $\frac{t^2 \cdot 3t^4}{4t^3} = \frac{3t^6}{4t^3} = \frac{3t^3}{4}$
21. $\frac{1}{4x^2}$
22. $\frac{1}{a^5}$
23. 1
24. t^2
25. $\frac{1}{c^9}$
26. s^{14}
27. $(\frac{c^{-6}}{c^{15}})^{-1} = (\frac{1}{c^{21}})^{-1} = c^{21}$
28. $\frac{1}{w^8}$
29. $\frac{3x}{2y^4}$
30. $\frac{5t^3}{3s^2}$
31. $\frac{-1}{2xy^2}$
32. $6t^4/(3t^{-3}) = 2t^{4-(-3)} = 2t^7$
33. $\frac{-3}{10x^3}$
34. $576\ x^6y^{12}z^{18}$
35. $\frac{9x^6}{y^8}\frac{x^{21}}{6^3y^{15}} = \frac{x^{27}}{24y^{23}}$
36. $\frac{10b}{3}$
37. $(-4p^{-4})(3p^{-3})/(3p^{-2}4p^5) = \frac{-12p^{-7}}{12p^3} = \frac{-1}{p^{10}}$
38. $\frac{15}{2x}$
39. $(\frac{y}{2x})^7 = \frac{y^7}{2^7x^7}$
40. $(\frac{y}{2x})^{-7} = (\frac{2x}{y})^7$
41. $\frac{8x^2}{15}$
42. $\frac{9a^4}{x^{10}}$
43. w^8
44. $\frac{1}{t^2}$
45. s^{34}
46. $\frac{3x^3}{2z^2}$
47. $\frac{3xy^4}{2}$
48. $\frac{2a^2}{5x^2}$
49. $\frac{6x^3}{5y^3}$
50. $\frac{-21a^2}{b^2c^4}$
51. $\frac{a^4b^4}{a^9b^{-9}} = \frac{b^{13}}{a^5}$
52. $\frac{a-2}{2a}$
53. $\frac{x^5y^2}{-5}$
54. $\frac{x}{3y}$
55. $\frac{25a^4b^{-6}}{1000a^{12}b^6} = \frac{1}{40a^8b^{12}}$
56. $\frac{10a^8b^7}{3}$
57. $(-2^{-2}xy^{-2}z^3)^{-3} = -2^6x^{-3}y^6z^{-9} = \frac{-64y^6}{x^3z^9}$
58. $(\frac{y^4}{-3x^3})^2(\frac{6y^5}{x^7})^3 = (\frac{y^8}{9x^6})(\frac{216y^{15}}{x^{21}}) = \frac{24y^{23}}{x^{27}}$
59. $6x^2y^{11}\ -10x^3$
60. $\frac{1}{3a(x+1)}$
61. 400
62. 160000
63. 40
64. 8000
65. 0.2

66. 0.04
67. 0.0016
68. 0.008
69. 0.16
70. 0.5
71. $2\sqrt{21}$
72. $14\sqrt{3}$
73. $10\sqrt{14}$
74. $6\sqrt{30}$
75. $30\sqrt{6}$
76. $12\sqrt{3}$
77. $5\sqrt{7}$
78. $4\sqrt{3}$
79. $5\sqrt{5}$
80. $3\sqrt{5}$
81. $12\sqrt{3}$
82. $5\sqrt{7}$
83. $4\sqrt{3}$
84. $5\sqrt{5}$
85. $3\sqrt{5}$
86. $6\sqrt{5}$
87. $22\sqrt{3}$
88. $15\sqrt{3}$
89. $11\sqrt{5}$
90. $5y\sqrt{3y}$
91. $(3x + 5)^2$
92. $(2x - 5y)^2$
93. $(4x - 3)(6x + 5)$
94. $(x + 5)(y - 8)$
95. $3(x + 2)(x - 2)$
96. $(x - 3)(y + 3)$
97. $(5x - y)(4x + 7y)$
98. $3y(y - 8)(y - 4)$
99. $5(3x + 7)(x + 2)$
100. $9xy(x - 3)$

Answer Key

1. -108
2. $\frac{1}{72}$
3. 3^7
4. 3^{14}
5. $(\frac{1}{3^3})^{-2} = 3^6$
6. $-x^6$
7. $-27y^3$
8. $16t^8$
9. $p^{-6}p^6 = p^0 = 1$
10. $x^{-1}\cdot x^2\cdot x^{-3}\cdot x^4\cdot x^{-5} = 1/x^3$
11. $1/x^{14}$
12. $15a^2x^4$
13. $-x^{14}y^{21}$
14. $\frac{24y^8}{x^5}$
15. $-189a^{10}b^4c^4$
16. $\frac{y^4}{x^2}$
17. $\frac{x^3}{3}$
18. $\frac{-5y^6}{24x^8}$
19. $\frac{9y^4}{4x^7}$
20. $\frac{-4y^3}{x^3}$
21. $\frac{2a^2}{5x^2}$
22. $(-2a^2\ x^{-5}) = -\frac{2a^2}{x^5}$
23. $(-2^{-2}xy^{-2}z^3)^{-3} = -2^6x^{-3}y^6z^{-9} = \frac{-64y^6}{x^3z^9}$
24. $\frac{6x^3}{5y^3}$
25. $\frac{8x^2}{15}$
26. w^8
27. $\frac{x^5y^2}{-5}$
28. $\frac{-21a^2}{b^2c^4}$
29. $\frac{a^4b^4}{a^9b^{-9}} = \frac{b^{13}}{a^5}$
30. $\frac{a-2}{2a}$
31. $\frac{3x^3}{2z^2}$
32. $(\frac{y^4}{-3x^3})^2(\frac{6y^5}{x^7})^3 = (\frac{y^8}{9x^6})(\frac{216y^{15}}{x^{21}}) = \frac{24y^{23}}{x^{27}}$
33. $\frac{x}{3y}$
34. $\frac{1}{3a(x+1)}$
35. $\frac{25a^4b^{-6}}{1000a^{12}b^6} = \frac{1}{40a^8b^{12}}$
36. $6x^2y^{11} - 10x^3$
37. $\frac{10a^8b^7}{3}$
38. $\frac{3xy^4}{2}$
39. $\frac{1}{t^2}$
40. s^{34}
41. $125t^6$
42. $-108x^{13}y^8$
43. $72x^{13}y^5z^9$
44. $-w^{10}$
45. $\frac{(12x^{-3}y^2)^2}{5x^5y^3}\cdot\frac{(10x^{-1}y^{-2})^3}{72x^{-3}y^{-4}}$
$= \frac{144x^{-6}y^4}{5x^5y^3}\cdot\frac{1000x^{-3}y^{-6}}{72x^{-3}y^{-4}}$
$= \frac{144y}{5x^{11}}\cdot\frac{1000}{72y^2}$
$= \frac{400}{x^{11}y}$
46. $\frac{6x^2y^{-3}}{7x^{-4}y^3}\cdot\frac{49x^{-1}y^4}{30x^3y^2}$
$= \frac{6x^6}{7y^6}\cdot\frac{49y^2}{30x^4}$
$= \frac{7x^2}{5y^4}$
47. $(\frac{12x^{-3}y^2}{5x^5y^3})^2$
$= (\frac{12}{5x^8y})^2$
$= \frac{144}{25x^{16}y^2}$
48. $(\frac{10x^{-1}y^{-2}}{3x^{-3}y^{-4}})^3$
$= (\frac{10x^2y^2}{3})^3$
$= \frac{1000x^6y^6}{27}$
49. $\frac{144}{25x^{16}y^2}\cdot\frac{1000x^6y^6}{27}$
$= \frac{640y^4}{3x^{10}}$
50. $\frac{25}{144x^{16}y^2}\div\frac{1000x^6y^6}{27}$
$= \frac{25}{144x^{16}y^2}\cdot\frac{27}{1000x^6y^6}$
$= \frac{3}{640x^{22}y^8}$
51. w^8

52. $(-2x^4y)/\{(-2x^{-2}y^{-2})(-5xy)\} = \frac{x^5y^2}{-5}$

53. $(7abc^{-4})(9a^5b^{-2})/(-3a^4b) = \frac{-21a^2}{b^2c^4}$

54. $(-2^{-2}xy^{-2}z^3)^{-3} = -2^6x^{-3}y^6z^{-9} = \frac{-64y^6}{x^3z^9}$

55. $\frac{a^4b^4}{a^9b^{-9}} = \frac{b^{13}}{a^5}$

56. $\frac{a-2}{2a}$

57. $\frac{3x^3}{2z^2}$

58. $(\frac{-3x^3}{y^4})^{-2}(\frac{x^7}{6y^5})^{-3} = (\frac{y^4}{-3x^3})^2(\frac{6y^5}{x^7})^3 =$
$(\frac{y^8}{9x^6})(\frac{216y^{15}}{x^{21}}) = \frac{24y^{23}}{x^{27}}$

59. $\frac{x^2y^3}{x^4y^2}\cdot(\frac{x^7y^4}{x^5y^6})^{-2}$
$= \frac{y}{x^2}\cdot(\frac{x^2}{y^2})^{-2}$
$= \frac{y}{x^2}\cdot\frac{y^4}{x^4}$
$= \frac{y^5}{x^6}$
$= -1/2$

60. $(xy^2)^{-3}\cdot(\frac{5x^{-1}y^4}{2x^3y^2})^{-2}$
$= \frac{1}{x^3y^6}\cdot(\frac{5y^2}{2x^4})^{-2}$
$= \frac{1}{x^3y^6}\cdot\frac{4x^8}{25y^4}$
$= \frac{4x^5}{25y^{10}}$
$= -4/25$

61. ± 5 since $5^2 = 25$ and $(-5)^2=25$.

62. 2 since $2^3 = 8$ (-2 is not).

63. ± 3 since $3^4 = 81$ and $(-3)^4=81$.

64. $n\sqrt{48n} - \sqrt{27n^3} = 4n\sqrt{3n} - 3n\sqrt{3n} = n\sqrt{3n}$

65. $\sqrt{x^2+4x+4} = |x+2|$

66. $9t^2$

67. 5 (-5 is a square of 25, but not the principal one.)

68. 2 is the principal cube root of 8

69. 3

70. $6x$

71. $\frac{2}{3}$

72. -2

73. 3

74. 9

75. -3

76. -1

77. 1

78. -1

79. $10y$

80. $\frac{2}{3}$

81. 0.0625

82. 0.064

83. 0.8

84. 0.4096

85. 0.8

86. 0.512

87. 0.09

88. 0.0081

89. 0.3

90. 0.027

91. $(2x - 9y)(x + 4y)$

92. $3(4x + 9)(2x - 3)$

93. $5(3x - 2)(2x + 5)$

94. $6(2x - 3)(x + 2)$

95. $8(3x - 4)(x + 1)$

96. $3x(x - 1)(x + 1)(x^2 + 1)$

97. $(x + 6y)^2$

98. $(2x - 7y)^2$

99. Note that $8 = 1\times8 = 2\times4$
$x^4 + 6x^2 + 8 = (x^2 + 2)(x^2 + 4)$

100. $(z^3 - 8)(z^3 + 1)$
$= (z - 2)(z^2 + 2z + 4)(z + 1)(z^2 - z + 1)$

Answer Key

1. 60 $5\times4\times3 = 60$
2. There are 15 possible candidates for the president. Once the president is elected, there are 14 possible choices for the vice president. Therefore, there are 15×14 possible outcomes for the election. We can also express $15\times14 = {}_{15}P_2$.
3. ${}_{10}P_4 = 10\times9\times8\times7 = 5040$
4. 9!
5. ${}_{25}P_2 = 25\times24 = 600$
6. There are $10\times10\times10\times26\times26\times26 = 17{,}576{,}000$ possible different plates.
7. $26\times26\times26\times10\times10\times26 = 45{,}697{,}600$ possible different plates.

		the first dice			
		1	2	3	4
the second dice	1	(1, 1)	(2, 1)	(3, 1)	(4, 1)
	2	(1, 2)	(2, 2)	(3, 2)	(4, 2)
	3	(1, 3)	(2, 3)	(3, 3)	(4, 3)
	4	(1, 4)	(2, 4)	(3, 4)	(4, 4)

8. There are 16 'outcomes', but be aware that all the non-shaded outcomes are duplicated. For example, (1, 2) and (2, 1) are the same since the order does not matter. We list them all because the repetition accounts for the number of occurrences by itself.
9. (4, 1), (3, 2), (2, 3), (1, 4) are such events, the probability is $\frac{4}{16} = \frac{1}{4}$
10. (2, 2), (2, 4), (4, 2) and (4, 4), the probability is $\frac{4}{16} = \frac{1}{4}$
11. (1, 1), (1, 3), (3, 1), and (3, 3), and it has a probability of $\frac{1}{4}$.
12. There are two ways of approach.
    Method I: Either the first die or the second dice is a even, the total count is 12, thus the probability is $\frac{12}{16} = \frac{3}{4}$.
    Method II: The events of 'product of a even number' is supplementary to 'product of an odd number', which has a probability of $\frac{1}{4}$ from the result of the previous problem. Therefore, the probability of getting an even product is $1 - \frac{1}{4} = \frac{3}{4}$.

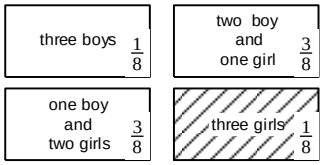
		the first dice			
		1	2	3	4
the second dice	1	(1, 1)	(2, 1)	(3, 1)	(4, 1)
	2	(1, 2)	(2, 2)	(3, 2)	(4, 2)
	3	(1, 3)	(2, 3)	(3, 3)	(4, 3)
	4	(1, 4)	(2, 4)	(3, 4)	(4, 4)

13. The outcomes of such an event has been shaded, and the probability is $\frac{3}{8}$.
14. 20:1
15. $\frac{20}{21}$
16. $\frac{1}{40}$
17. 39:1
18. Since 3, and 6 are the only two 3-multiples, the odds against rolling a 3-multiple is 4:2.
19. $\frac{8}{9}$, Becky has a greater chance of winning the tournament.
20. $\frac{1}{9}$
21. $\frac{6}{36} = \frac{1}{6}$ All the cells with odd product are shaded.

		the first dice					
		1	2	3	4	5	6
the second dice	1	1	2	3	4	5	6
	2	2	4	6	8	10	12
	3	3	6	9	12	15	18
	4	4	8	12	16	20	24
	5	5	10	15	20	25	30
	6	6	12	18	24	30	36

22. The probability for a 2 (or any number 1-6) is $\frac{1}{6}$, thus the probability to roll a 2 or 3 is $\frac{1}{6}\times2 = \frac{1}{3}$.
23. The set of odd numbers = {1, 3, 5}, the set of the numbers greater than 2 = {3, 4, 5, 6}. The union of two sets is {1, 3, 4, 5, 6}. The probability of getting an odd number or a number greater than 2 is $5\times\frac{1}{6} = \frac{5}{6}$.
24. The set of numbers greater than 5 = {6}. The set of numbers less than 3 = {1, 2}. The union of two sets is {1, 2, 6}. The probability of rolling a number greater than 5 or less than 3 is $3\times\frac{1}{6}=\frac{1}{2}$.
25. A = {4, 5, 6}, B = {1, 2, 3, 4}. $A\cup B$ = {1, 2, 3, 4, 5, 6}. Thus, the probability = $6\times\frac{1}{6} = 1$.
26. There are 4 aces and 4 kings. The probability = $8\times\frac{1}{52}=\frac{2}{13}$.

27. There are 13 hearts and 3 kings that are not heart. Thus, there are $16\times\frac{1}{52}=\frac{4}{13}$.
28. J, Q, K are picture cards. There are 12 of them. There are 26 black cards. However, the overlapping are black cards with picture. There are 6 of them. Thus, there are 32 cards witch picture or black color. The probability is $32\times\frac{1}{52}=\frac{8}{13}$.
29. There are 13 clubs and 26 red cards. There is no intersection. Thus, the probability is $\frac{3}{4}$.
30. There are 7×3=21 cards less than 8, which are not diamond. There are 13 diamonds. Thus, there are 21+13=34 cards less than 8 or a diamond.
31. There are 4×2=8 red cards that are greater than 9. There are 13×2=26 black cards. Thus, the probability is $(8+26)\times\frac{1}{52}=\frac{17}{26}$.
32. *P*(the youngest child is a girl) = $\frac{1}{2}$, *P*(the middle child is a boy) = $\frac{1}{2}$ and *P*(the oldest child is a girl) = $\frac{1}{2}$. Thus, the probability of the specified event = $(\frac{1}{2})^3=\frac{1}{8}$.
33. $(\frac{1}{2})^3=\frac{1}{8}$
34. $(\frac{1}{2})^3=\frac{1}{8}$
35. $\frac{1}{2}\times(\frac{1}{2})^2=\frac{1}{8}$
36. There are three cases: (a) the oldest is a boy, and the rest are girls; (b) the middle is a boy, and the rest are girls; (c) the youngest is a boy, and the rest are girls. Now that *P*(case a) = $\frac{1}{8}$. Similarly, *P*(case b) = *P*(case c) = $\frac{1}{8}$. Therefore, *P*(only one boy) = $\frac{3}{8}$.
37. What is the opposite case of "at least one boy?" As illustrated below, the unshaded boxes represent "at least one boy." It is "no boy." The *P*(no boy) = *P*(three girls) = $\frac{1}{8}$. (We have done it in Problem 12.) Thus, *P*(at one boy) = 1 - $\frac{1}{8}$ = $\frac{7}{8}$.



38. $\frac{2}{4}\times\frac{1}{4}=\frac{1}{8}$
39. $\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$
40. For the first draw, *P*(not red) = 1 - *P*(a red) = 1 - $\frac{1}{2}$ = $\frac{1}{2}$. Thus, *P*(no red for two draws) = *P*(first not red)·*P*(second not red) = $\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$.
41. For the first draw, *P*(not green) = 1 - *P*(a green) = 1 - $\frac{1}{4}$ = $\frac{3}{4}$. Thus, *P*(no green for two draws) = *P*(first not green)·*P*(second not green) = $\frac{3}{4}\times\frac{3}{4}=\frac{9}{16}$.
42. There are 8 cards, the number of outcomes should be ${}_8P_4$. However, the actual number is less since there are 2 'A's and 2 'E's. The actual number after adjustment is
$\frac{{}_8P_4}{2!2!}=\frac{8\cdot7\cdot6\cdot5}{2\cdot2}=420$.
43. 'W', 'E' and 'T' has 6 permutation outcomes. Except 'E', there are three cards 'A', 'H', and 'R' can be selected and arranged as below:
*x*yyy, y*x*yy, yy*x*y, yyy*x*
x can be one of {A, H, R}, each y can take away one of {W, E, T}. There are 3×4×6 = 72 different such outcomes.
For 'E', we have
EEyy, yEEy, yyEE,
EyEy, EyyE, yEyE
each y can take a way {W, T}, there are 6×2 = 12 outcomes.
There are 72+12 = 84 outcomes.
44. Let's fix the three cards in the order WET, waiting for one more card to complete.
The result can be
*x*WET, W*x*ET, WE*x*T, WET*x*,
where *x* can be the result of the cards: 'A', 'H', and 'R', which has 12 outcomes or
EWET, WEET, WETE
which has 3 outcomes. Therefore, the total number of outcomes is 15.
45. 1234, 1243, 1324, 1342, 1423, 1432,
2134, 2143, 2314, 2341, 2413, 2431,
3124, 3142, 3214, 3241, 3412, 3421,
4123, 4132, 4213, 4231, 4312, 4321
46. 6 3! = 3×2×1 = 6
47. 24 4! = 4×3×2×1 = 24
48. 12 ${}_4P_2 = 4\times3 = 12$
49. True ${}_5P_5 = 5\times4\times3\times2\times1$
50. True
51. False
52. True
53. False
54. False
55. 3628800 10! = 3628800 different patterns.
56. 5040 7! = 5040 different designed patterns.
57. 720 The first prize can go to one of the 10 students, the second prize can go to one of the remaining 9 students, and the third prize can go to the remaining one of the remaining 8students. Therefore, there are 10×9×8 (or ${}_{10}P_3$ in short) = 720 different arrangements.
58. 120 How many different outcomes of selecting 3 students of out 10? It is ${}_{10}C_3=\frac{10\cdot9\cdot8}{1\cdot2\cdot3}=120$ different ways.

59. 120 ${}_{10}C_3 = \frac{10\cdot 9\cdot 8}{1\cdot 2\cdot 3} = 120$

60. 60 ${}_5P_3 = 5\times 4\times 3 = 60$

61. 7,200 The total of different ways of giving 3 (different) books to 3 students (among 5) is $120\times 60 = 7{,}200$.

62. P(the first is red)×P(the second is blue [under the condition that only three marbles are left since one (red) is drawn]) = $\frac{2}{4}\times\frac{1}{3}=\frac{1}{6}$.

63. P(first blue)×P(second blue) = $\frac{1}{4}\times 0 = 0$.

64. P(first not red)×P(second not red [under the condition that only three marbles are left since one (blue or green) is drawn]) = $\frac{1}{2}\times\frac{1}{3} = \frac{1}{6}$

65. P(no green marbles) = P(first not green)×P(second not green [under the condition that only three marbles are left since one (red or blue) is drawn.]) = $\frac{3}{4}\times\frac{2}{3}= \frac{1}{2}$.

66. P(the first favors Alex)×P(the second favors Bryan)×P(the third favors Charlie) = $\frac{50}{150}\times\frac{65}{149}\times\frac{20}{148}=\frac{325}{16539}$.

67. P(the first two has no opinion)×P(the third favors Charlie) = $\frac{15}{150}\times\frac{14}{149}\times\frac{20}{148}=\frac{7}{5513}$.

68. P(no one favors Charlie) = $\frac{130}{150}\times\frac{129}{149}\times\frac{128}{148} = .648$

69. P(All three favor Alex) = $\frac{50}{150}\times\frac{49}{149}\times\frac{48}{148} = \frac{196}{5513}$

70. The failure rate of each of the torpedo is 0.6. Thus, both torpedo fails at a probability of 0.6×0.6=0.36.

71. P(both hit)= P(the first hits)×P(the second one hits[under the condition that the first one hits]) = 0.4×0.9=0.36.

72. P(the first hits, the second misses) = P(the first hits)×P(the second one misses [under the condition that the first one hits]) = 0.4×0.1=0.04.

73. P(the first one misses, the second one hits) = P(the first one misses)×P(the second one hits) = 0.6×0.4=0.24.

74. No They are dependent. Say, Alex (eldest), Ben, and Christ (youngest) are brothers. If Ben has the affliction, then Christ is more likely to get the defect than Alex.

75. 0.999

76. P(both have the affliction) = P(the first has)×P(the second one also has) = 0.001×0.04=0.00004

77. P(the first has, the second has not) = P(the first has)×P(the second has not [under the condition that the first one has]) = 0.001×0.96 = 0.00096

78. P(both do not have) = P(the first does not have)×P(the second does not have [under the condition that the first one does not have]) = 0.999×0.999 = $(1-0.001)^2$ = 0.998001