

Answer Key

1. 5
2. 8
3. 11
4. 2
5. 8
6. 14
7. 5
8. 2
9. 8
10. 24
11. 16
12. 2
13. 18
14. 4
15. 36
16. 25
17. 2
18. 12
19. 6
20. 3
21. 15
22. 3
23. 27
24. 6
25. 35
26. 5
27. 9
28. 6
29. 54
30. 9
31. $12 - 5 = 7$
32. $30 \times 3 = 90$
33. $21 \div 7 = 3$ (wks)
 $3 \times 2 = 6$ pounds
34. $800 - 600 = 200$
 $200 \div 5 = 40$
35. $400 - 300 = 100$
 $100 \div 2 = 50$ (calf)
 $300 + 50 = 350$ (kg, cow)
36. $1 + 2 + 4 = 7$
 $28 \div 7 = 4$
 $1 \times 4 = 4$ (3rd)
 $2 \times 4 = 8$ (2nd)
 $4 \times 4 = 16$ (1st)
 $4 + 8 + 16 = 28$ (double-check)
Ans = 16 8 4 (Listed in order separated by space or comma)
37. $43 \div 3 = \underline{14}$ R1
Ans = 14 (pcs)
38. $35 \times 6 = \$210$
39. $0.3 \times 7 = \$2.10$
40. $8 \div 40 = 0.20 = 20 \text{ ¢}$
41. $75 \div 15 = 5$ plates
42. $392 \div 7 = 56$
43. $86 - 59 = \$27$
44. $15 \div 5 = 3$ pieces
45. $9 \times 2 \times 20 = 360$ rooms
46. $2 \times 0.85 = 1.70$
 $3 \times 0.90 = 2.70$
 $1.70 + 2.70 = 4.40$
 $5 - 4.40 = \$0.60$
47. $2 \times 24 + 14 = 62$
48. (a) $9:30 - 1:00 = 8:30$ P.M.
(b) 9:30 A.M.
49. $13:15 - 1:30 = 11:45$ A.M.
50. 16
51. $12.60 \times \frac{1}{2} = 6.30$
 $6.30 \times 5 = \$31.50$
52. D: $14 - 8 = 6$
C: $6 \div 2 = 3$
B: $10 - 3 = 7$
Ans = 7 B & 3 C and 6 D
53. 1 (Add $\frac{1}{4}$.)
54. $2 \times 26.5 = 53$
 $66 - 53 = 13$
55. $3 \times 6 = \underline{18 \text{ ft}}$
56. D
They all have the same number of pencils.
B: $\frac{1}{3}$
C: $\frac{1}{2}(1 - \frac{1}{3}) = \frac{1}{3}$
A: $1 - \frac{1}{3} - \frac{1}{3} = \frac{1}{3}$

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57. A
Andrea has the least.
B: $\frac{1}{2}$
C: $\frac{1}{3}(1 - \frac{1}{2}) = \frac{1}{6}$
A: $1 - \frac{1}{2} - \frac{1}{6} = \frac{1}{3}$
58. $\frac{2}{3} \times 18 = 12$
 $12 \times 15 = \$180$
59. $4 \times \frac{1}{8} = \frac{1}{2}$
60. $\frac{1}{3} \times 24 = 8$
 $24 - 8 = 16$
61. C
62. $18 - 9 = 9$
 $36 - 9 = \underline{27}$
63. $6 \times 5 \times 4 = \underline{120 \text{ hours}}$
64. 1 more softball
65. David: $89 + 4 = 93$
Ashley: $\frac{1}{3} \times 93 = 31$
Stephen: $2 \times 31 = \underline{62 \text{ pounds}}$
66. B
67. $(3 + 4 + 5) \div 3 = 4$
68. $(13 + 15) \div 2 = 14$
69. $(26 + 20) \div 2 = 23$
70. $(80 + 100 + 95 + 85) \div 4 = 90$

Answer Key

- | | | | |
|--|--|---------------------|---------------------|
| 1. $11\frac{1}{4}$ | 22. $9\frac{2}{3}$
$29 \div 3 = 9 \text{ r } 2$ | 44. $\frac{1}{9}$ | 74. $\frac{6}{7}$ |
| 2. $10\frac{1}{6}$ | 23. $9\frac{1}{3}$
$28 \div 3 = 9 \text{ r } 1$ | 45. $1\frac{2}{4}$ | 75. $\frac{6}{11}$ |
| 3. $11\frac{1}{8}$ | 24. $6\frac{1}{4}$
$25 \div 4 = 6 \text{ r } 1$ | 46. $\frac{2}{7}$ | 76. $\frac{8}{9}$ |
| 4. $8\frac{1}{9}$ | 25. $7\frac{1}{4}$
$29 \div 4 = 7 \text{ r } 1$ | 47. $1\frac{3}{10}$ | 77. $\frac{1}{7}$ |
| 5. $7\frac{1}{6}$ | 26. $8\frac{3}{4}$
$35 \div 4 = 8 \text{ r } 3$ | 48. $\frac{3}{6}$ | 78. $\frac{1}{5}$ |
| 6. $10\frac{2}{4}$ | 27. $5\frac{1}{5}$
$26 \div 5 = 5 \text{ r } 1$ | 49. $\frac{4}{6}$ | 79. $\frac{1}{4}$ |
| 7. $10\frac{2}{6}$ | 28. $5\frac{3}{5}$
$28 \div 5 = 5 \text{ r } 3$ | 50. $1\frac{4}{10}$ | 80. $\frac{9}{11}$ |
| 8. $10\frac{3}{8}$ | 29. $6\frac{1}{6}$
$37 \div 6 = 6 \text{ r } 1$ | 51. 25 | 81. 10.5 |
| 9. $8\frac{1}{9}$ | 30. $7\frac{1}{10}$
$71 \div 10 = 7 \text{ r } 1$ | 52. 28 | 82. 7.5 |
| 10. $6\frac{1}{6}$ | 31. $11\frac{1}{4}$ | 53. 28 | 83. 5.5 |
| 11. $\frac{22}{3}$
$7 \times 3 + 1 = 22$ | 32. $10\frac{1}{6}$ | 54. 21 | 84. 6.5 |
| 12. $\frac{37}{5}$
$7 \times 5 + 2 = 37$ | 33. $11\frac{1}{8}$ | 55. 16 | 85. 3.5 |
| 13. $\frac{13}{4}$
$3 \times 4 + 1 = 13$ | 34. $8\frac{1}{9}$ | 56. 36 | 86. 1.75 |
| 14. $\frac{19}{4}$
$4 \times 4 + 3 = 19$ | 35. $7\frac{1}{6}$ | 57. 28 | 87. 1.5 |
| 15. $\frac{35}{3}$
$11 \times 3 + 2 = 35$ | 36. $10\frac{2}{4}$ | 58. 72 | 88. 6.75 |
| 16. $\frac{15}{4}$
$3 \times 4 + 3 = 15$ | 37. $10\frac{2}{6}$ | 59. 21 | 89. 10.75 |
| 17. $\frac{27}{5}$
$5 \times 5 + 2 = 27$ | 38. $10\frac{3}{8}$ | 60. 20 | 90. 4.25 |
| 18. $\frac{43}{6}$
$7 \times 6 + 1 = 43$ | 39. $8\frac{1}{9}$ | 61. $\frac{2}{3}$ | 91. $8\frac{1}{10}$ |
| 19. $\frac{24}{7}$
$3 \times 7 + 3 = 24$ | 40. $6\frac{1}{6}$ | 62. $\frac{1}{9}$ | 92. $5\frac{7}{10}$ |
| 20. $\frac{35}{8}$
$4 \times 8 + 3 = 35$ | 41. $1\frac{4}{5}$ | 63. $\frac{7}{10}$ | 93. $1\frac{1}{10}$ |
| 21. $6\frac{1}{3}$
$19 \div 3 = 6 \text{ r } 1$ | 42. $\frac{1}{3}$ | 64. $\frac{7}{11}$ | 94. $7\frac{2}{5}$ |
| | 43. $1\frac{4}{5}$ | 65. $\frac{8}{11}$ | 95. $2\frac{1}{4}$ |
| | | 66. $\frac{7}{8}$ | 96. $4\frac{3}{4}$ |
| | | 67. $\frac{1}{8}$ | 97. $10\frac{1}{8}$ |
| | | 68. $\frac{1}{3}$ | 98. $8\frac{1}{8}$ |
| | | 69. $\frac{1}{6}$ | 99. $2\frac{3}{8}$ |
| | | 70. $\frac{2}{5}$ | 100. $8\frac{7}{8}$ |
| | | 71. $\frac{7}{9}$ | |
| | | 72. $\frac{5}{9}$ | |
| | | 73. $\frac{1}{2}$ | |

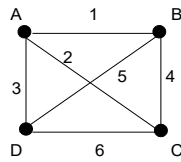
Answer Key

1. $\frac{1}{15}$
2. $\frac{11}{12}$
3. $\frac{13}{72}$
4. $\frac{2}{45}$
5. $\frac{29}{144}$
6. $\frac{34}{75}$
7. $\frac{41}{45}$
8. $\frac{49}{96}$
9. $\frac{67}{144}$
10. $\frac{8}{105}$
11. $4/5$
12. $7/9$
13. $4/9$
14. $6/7$
15. $9/8$
16. $2/9$
17. $5/9$
18. $5/6$
19. $2/3$
20. $3/5$
21. $\frac{4}{8}$
Hint: $\frac{2}{3}$
22. $\frac{15}{21}$
Hint: $\frac{4}{7}$
23. $\frac{24}{32}$
Hint: $\frac{6}{7}$
24. $\frac{18}{12}$
Hint: $\frac{5}{4}$
25. $\frac{8}{24}$
Hint: $\frac{1}{6}$
26. $\frac{24}{30}$
Hint: $\frac{8}{9}$
27. $\frac{6}{9}$
Hint: $\frac{5}{9}$
28. $\frac{16}{12}$
Hint: $\frac{7}{6}$
29. $\frac{24}{16}$
Hint: $\frac{5}{4}$
30. $\frac{2}{3}$
Hint: $\frac{1}{3}$
31. 10.5
32. 7.5
33. 5.5
34. 6.5
35. 3.5
36. 1.75
37. 1.5
38. 6.75
39. 10.75
40. 4.25
41. 25
42. 15
43. 12
44. 40
45. 9
46. 12
47. 24
48. 48
49. 15
50. 36
51. 74
52. 82
53. 88
54. 92
55. 87
56. 93
57. 86
58. 94
59. 100
60. 104
61. $\frac{2}{3}$
62. $\frac{4}{7}$
63. $\frac{4}{9}$
64. $\frac{5}{18}$
65. $\frac{24}{25}$
66. $\frac{3}{32}$
67. $\frac{7}{45}$
68. $\frac{21}{64}$
69. $\frac{1}{2}$
70. $\frac{2}{5}$
71. $0.3 = \frac{3}{10}$
72. $\frac{25}{100} = \frac{1}{4}$
73. $5.5 = 5\frac{5}{10}$
reduce $\rightarrow 5\frac{1}{2} = 5\frac{1}{2}$
74. $1.25 = 1\frac{25}{100}$
reduce $\rightarrow 1\frac{5}{20}$
reduce $\rightarrow 1\frac{1}{4} = 1\frac{1}{4}$
75. $\frac{35}{100} = \frac{7}{20} = 7/20$
76. $0.2 = \frac{1}{5} = 1/5$
77. $0.7 = \frac{7}{10} = 7/10$
78. $0.4 = \frac{2}{5} = 2/5$
79. $0.9 = \frac{9}{10} = 9/10$
80. $3.75 = 3\frac{3}{4} = 3\frac{3}{4}$
81. 6.25
82. 50
83. 700
84. 3500
85. 25
86. 3.125
87. 25
88. 0.016
89. 0.25
90. 100
91. $1\frac{1}{6}$
92. $8/21$
93. 25
94. 20
95. $1/6$
96. $\left(\frac{14}{5}\right)\left(\frac{15}{7}\right) = 6$
97. $\left(\frac{18}{7}\right)\left(\frac{14}{9}\right) = 4$
98. $\left(\frac{15}{4}\right)\left(\frac{12}{5}\right) = 9$
99. 22
100. 10
101. $6\frac{1}{4} - 3\frac{1}{2} = 2\frac{3}{4}$ ft
102. $2 \times .75 = 1.5$
 $4 \times .75 = 3$
 $6 \times .75 = 4.50$
 $7 \times .75 = 5.25$
At most 6 cans.
103. $36 + 24 = 60$
 $720 \div 60 = 12$ pairs
 $12 \times 2 =$ 24 (cars)
104. $720 \div 2 = 360$
 $360 \div 36 = 10$
 $360 \div 24 = 15$
 $10 + 15 =$ 25 (cars)
105. $65 + 4 = 69$
106. $60 + 64 = 124$
107. $100 \div 2 = 50$
 $4 \div 2 = 2$
 $50 + 2 = 52$
(grandpa)
 $50 - 2 = 48$
(grandma)
108. $70 + 74 = 144$
109. 61
110. $130 \div 2 = 65$
 $4 \div 2 = 2$
 $65 + 2 = 67$
(grandpa)
 $65 - 2 = 63$
(grandma)
111. $3 \times 15 = 45$
112. $45 - 20 - 8 = 17$
113. 13, 15, 17
114. 10, 15, 20
115. C = 12, D = 15, E = 18

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116. 6 matches

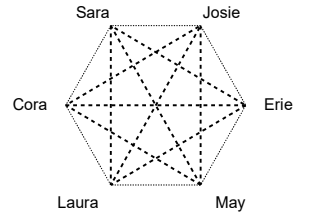
Assume they are A, B, C, and D. There are 6 matches as below:
AB, AC, AD, BC, BD, and CD.



117. 6 segments

120. 15 skits

Count the number of segments in the following diagram.

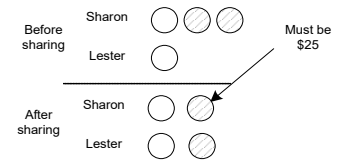


118. 10 segments

119. 15 matches

Answer Key

1. 3
2. -3
3. 25
4. 2
5. -5
6. -4
7. -1
8. -2
9. 14
10. 17
11. -23
12. 1
13. 3
14. 4
15. 27
16. -27
17. 2
18. 7
19. -18
20. -27
21. -2
22. 21
23. 27
24. 6
25. 8
26. 5
27. -3
28. -2
29. 6
30. -4
31. A
32. C
33. B
34. D
35. B
36. C
37. D
38. B
39. D
40. C
41. $\frac{2}{5}$
42. $\frac{1}{8}$
43. $1\frac{1}{4}$
44. $3\frac{1}{4}$
45. $3\frac{3}{4}$
46. 125%
47. 120%
48. 225%
49. 375%
50. 404%
51. $\frac{5}{24}$
52. $\frac{3}{20}$
53. $\frac{4}{27}$
54. $\frac{7}{10}$
55. $\frac{1}{24}$
56. $\frac{1}{36}$
57. $\frac{11}{90}$
58. $\frac{13}{108}$
59. $\frac{1}{15}$
60. $\frac{8}{45}$
61. -9
62. 3
63. 5
64. -4
65. 4
66. 6
67. 4
68. -5
69. 6
70. 5
71. 16
72. 40
73. 45
74. 54
75. -12
76. -7
77. 6
78. -8
79. 96
80. -4
81. Boys:Girls = 12:8 = 3:2 (reduced)
82. 120:144 = 10:12 = 5:6
83. $8 \div 2 = 4$
 $5 \times 4 = 20$ grams
84. $2.7 \times 5\frac{1}{3}$
 $= 2.7 \times 5 + 2.7 \times \frac{1}{3}$
 $= 13.5 + 0.9$
 $= 14.4$
85. $42.75 \div 3 = \$14.25$
86. $184 \div 8 = \boxed{23 \text{ (miles)}}$
87. $36 \div 4 = 9$ pounds
88. 40:15 = 8:3
89. $144 \div 24 = 6$ gal
90. $25 \times 5 = 125$ mi
91. $6 \times \frac{4}{8} = 3$ m
92. $46 \div 4\frac{1}{2} = \$8$ per hour
93. $105 \div 5\frac{1}{4} = 20$ miles per gallon
94. D
95. -4
96. $13 - 5 = 8$
 $8 \div 4 = 2$
97. 16.8
98. $27.50 \times 2 - 2.00 + 2.50 + 17.25$
 $= 55 - 2 + 2.5 + 17.25$
 $= 55.5 + 17.25$
 $= \$72.75$
99. \$75 for Sharon & \$25 for Lester
(See the following figure.)



100. $29 - 5 - 4 - 2 = 18$
 $18 \div 3 = 6$
(a) $6 + 5 = 11$ (first)
(b) $6 + 4 = 10$ (second)
(c) $6 + 2 = 8$ (third)

Answer Key

- | | | | |
|--|---------------------------|---|--------------------------------|
| 1. $x = -\frac{3}{2}$ | 6. $x = -4$ | 11. 4.9×10^9 | 16. 4.9×10^{-7} |
| 2. $x = 5$ | 7. $x = 1$ | 12. 9×10^{10} | 17. 9×10^{-10} |
| 3. $x = -2$ | 8. $-1/3$ | 13. 6.4×10^9 | 18. 6.4×10^{-11} |
| 4. $x = -5$ | 9. $x = -3$ | 14. 1.25×10^{11} | 19. 1.25×10^{-10} |
| 5. $x = 1$ | 10. $x = -3$ | 15. 1.6×10^{21} | 20. 2.5×10^{-11} |
| 21. 1.3×10^{-5} | 27. 6.25×10^3 | 33. $3x^2 - 15x + 15$ | 39. $4x^2 - 9$ |
| 22. 2×10^2 | 28. 7×10^9 | 34. $4x^2 - 16x - 20$ | 40. $-2x^2 - 5x + \frac{8}{9}$ |
| 23. 2.5×10^{-11} | 29. 3.75×10^{-4} | 35. $-0.5x^2 - 1.5x + 20$ | |
| 24. 3×10^{-13} | 30. 4×10^{-5} | 36. $\frac{1}{3}x^2 - x - \frac{10}{3}$ | |
| 25. 5×10^{-11} | 31. $x^2 + 4x - 45$ | 37. $9x^2 - 81x + 180$ | |
| 26. 5×10^{-8} | 32. $x^2 - 7x + 10$ | 38. $2x^2 - 15x + 27$ | |
| 41. -6, 9 | 46. -40, 400 | 51. -10, 25 | 56. -8, 16 |
| 42. 3.6, 1.08 | 47. 1.4, 0.49 | 52. 0.8, 0.16 | 57. 1, 0.25 |
| 43. $4, (2x - 3)^2$ | 48. $(2x - 1)^2$ | 53. $(2x - 5)^2$ | 58. $(2x - 1)^2$ |
| 44. $24, (3x + 4)^2$ | 49. $(3x + 3)^2$ | 54. $(3x - 3)^2$ | 59. $(3x + 5)^2$ |
| 45. $25, (4x - 5)^2$ | 50. $49, (4x - 7)^2$ | 55. $1, (4x - 1)^2$ | 60. $9, (4x - 3)^2$ |
| 61. 3 & -6 | 66. $x = 58^\circ$ | 71. $x=40, y=22.5$ | 76. A |
| 62. $1\frac{1}{6}$ & $\frac{1}{6}$ | 67. 55° | 72. 70. | 77. D |
| 63. The answer is listed | 68. 100° | 73. 90° | 78. D |
| 64. $\angle 8 = \angle 10 = 130^\circ$ | 69. 15° | 74. B | 79. 35 |
| 65. $y = 68x = 112^\circ$ (| 70. 29° | 75. B | 80. 480 |
| 81. C | 86. 12 | 91. C | 96. A |
| 82. A | 87. 18 | 92. A | 97. C |
| 83. B | 88. B | 93. 161 | 98. B |
| 84. B | 89. A | 94. 17 | 99. D |
| 85. B | 90. D | 95. D | 100. 66 |

Answer Key

Statement	Reason
$AB = CD$	Given
$\text{arc}(BA) = \text{arc}(CD)$	$AB = CD$ and Theorem 1
$\text{arc}(BAC) = \text{arc}(ACD)$	$\text{arc}(BAC) = \text{arc}(BA) + \text{arc}(AC)$ $\text{arc}(ACD) = \text{arc}(CD) + \text{arc}(AC)$
$BC = AD$	$\text{arc}(BAC) = \text{arc}(ACD)$ and Theorem 1

1.

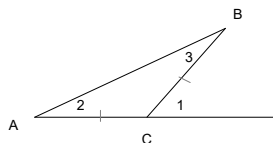
Statement	Reason
$BC = AD$	Given
$\text{arc} BAC = \text{arc} ACD$	$BC = AD$ and theorem 1
$\text{arc} BA = \text{arc} CD$	$\text{arc} BA = \text{arc} BAC - \text{arc} AC$ $\text{arc} CD = \text{arc} ACD - \text{arc} AC$
$BA = CD$	$\text{arc} BA = \text{arc} CD$ and

2.

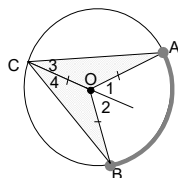
Statement	Reason
$\text{arc} BAC = \text{arc} ACD$	Given (both BC and AD are diameters)
$\text{arc} AB = \text{arc} CD$	$\text{arc} AB = \text{arc} BAC - \text{arc} AC$ $\text{arc} CD = \text{arc} ACD - \text{arc} AC$
$AB = CD$	$\text{arc} AB = \text{arc} CD$ and by theorem 1

3.

4. Since $\triangle ABC$ is an isosceles triangle. $\angle 2 = \angle 3$. Since $\angle 1$ is supplementary to $\angle ACB$, so is $\angle 2 + \angle 3$. Thus, $\angle 1 = \angle 2 + \angle 3 = 2\angle 2$.



5. Since $\triangle AOC$ and $\triangle COB$ are both isosceles triangles. With the result of previous problem, $\angle 1 = 2\angle 3$ and $\angle 2 = 2\angle 4$ have that $\angle AOB = \angle 1 + \angle 2 = 2(\angle 3 + \angle 4) = 2\angle ACB$. Thus, we have proved that the central angle is twice an inscribed angle.



6. $x = \frac{1}{2}(40^\circ) = 20^\circ$. $\angle BAD = 50^\circ = \frac{1}{2}(\text{arc } BCD)$. Thus, arc $BCD = 100^\circ$, hence $y = 60$.
7. $\angle ABC = \frac{1}{2}(\text{arc } ADC) = \frac{1}{2}(180^\circ) = 90^\circ$.

8. $\angle A = \frac{1}{2}\text{arc}(BC)$, $\angle B = \frac{1}{2}\text{arc}(AC)$, $\angle C = \frac{1}{2}\text{arc}(AB)$.
The sum of interior angles $= \angle A + \angle B + \angle C = \frac{1}{2}[\text{arc}(AB) + \text{arc}(AC) + \text{arc}(BC)] = \frac{1}{2}(\text{full circle}) = \frac{1}{2}(360^\circ) = 180^\circ$.

9. $a^\circ = \angle DAB = \frac{1}{2}(\text{arc } DCB)$ since the measure an inscribed angle is half of its circular angle. Similarly, $c^\circ = \angle BCD = \frac{1}{2}(\text{arc } BAD)$.
Thus, $a^\circ + c^\circ = \frac{1}{2}(\text{arc } DCB) + \frac{1}{2}(\text{arc } BAD) = \frac{1}{2}(360^\circ) = 180^\circ$.

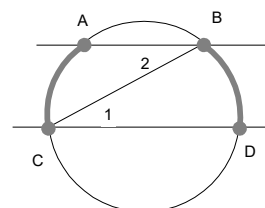
We can prove that $b^\circ + d^\circ = 180^\circ$ in the same way.

10. $\angle C : \angle A : \angle B = 1 : 3 : 5$. Thus,
 $\angle C = \frac{1}{9}(180^\circ) = 20^\circ$
 $\angle A = \frac{3}{9}(180^\circ) = 60^\circ$
 $\angle B = \frac{5}{9}(180^\circ) = 100^\circ$

11. 60°

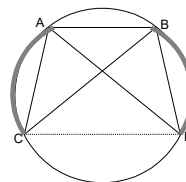
12. Left as exercise.

13. Since $AB \parallel CD$, $\angle 1 = \angle 2$. Now that $\text{arc}(AB) = 2\angle 1 = 2\angle 2 = \text{arc}(BD)$,



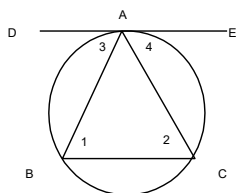
14. Reverse the arguments of the previous problem.

15. Since $\text{arc}(CAB) = \text{arc}(ABD)$
 $\text{arc}(CA)$
 $= \text{arc}(CAB) - \text{arc}(AB)$
 $= \text{arc}(ABD) - \text{arc}(AB)$
 $= \text{arc}(BD) \Rightarrow AB \parallel CD$

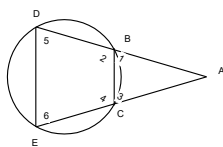


Geo+Alg (T1) Issue 5

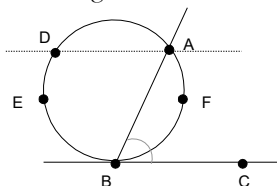
16. Our goal is to prove that $\angle 1 = \angle 2$. But how? Note that $\angle 2 = \angle 4$ (alternate interior angle) and $\angle 4 = \frac{1}{2}(\text{arc AC})$. We may conclude that $\angle 2 = \frac{1}{2}(\text{arc AC})$. Similarly, we have $\angle 1 = \frac{1}{2}(\text{arc AB})$. Since $DE \parallel BC$ (given), $\text{arc AB} = \text{arc AC}$, it follows that $\angle 1 = \angle 2$.



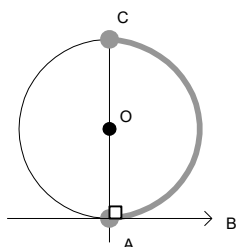
17. $\angle 1 + \angle 2 = 180$, $\angle 6 + \angle 2 = 180 \Rightarrow \angle 1 = \angle 6$, $\angle 3 = \angle 5$ (Similarly). Since $\angle 1 = \angle 3$ (Given), we have $\angle 5 = \angle 6 \Rightarrow \triangle ADE$ is an isosceles triangle.



18. $\angle 1 = \angle 5 = \frac{1}{2}(180 - \angle A)$, we conclude that $BC \parallel DE$ (congruent corresponding angle).
19. Since $\text{arc BC} = \text{arc CD}$, $\text{arc BD} = 2(\text{arc BC})$, $\angle OAD = \frac{1}{2}(\text{arc BD}) = \text{arc BC}$, $\angle BOC = \text{arc BC}$. Therefore, it follows that $\angle OAD = \angle BOC$ and thus $OC \parallel AD$ (congruent alternate interior).
20. $\angle BOC = \angle OAD$, $\text{arc BC} = \angle BOC$, $\text{arc BD} = 2\angle OAD$, therefore $\text{arc BC} = \text{arc CD}$.
21. Draw an auxiliary (dashed) line AD parallel to BC, hence, $\text{arc}(DEB) = \text{arc}(AFB) = x$. $\angle ABC = \angle BAD$ (alternate interior angle) $= \frac{1}{2}\text{arc}(BED) = \frac{1}{2}x$, by inscribed angle theorem.



22. $\angle BAC = \frac{1}{2}\text{arc}(AC) = \frac{1}{2}(180^\circ) = 90^\circ$. Thus, we have proved $OA \perp AB$.



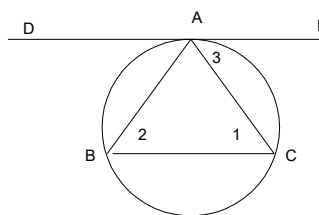
23. Draw a diameter CD parallel to AB. Since $\text{arc}(AC) = \text{arc}(AD)$, by parallel chord theorem, it follows that

$$\text{arc}(AC) = \text{arc}(AD) = 90^\circ$$

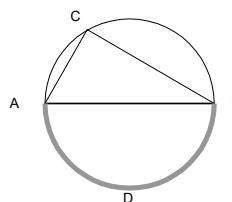
$$\angle AOC = \angle AOD = 90^\circ$$

$$\angle BAO = \angle AOC = 90^\circ \text{ (alternate interior angle)}$$

24. $\angle BAC = \frac{1}{2}(360^\circ - x^\circ - x^\circ) = 180^\circ - x^\circ$
25. The measure of an inscribed angle is half of the corresponding arc (central angle). Therefore, $x = \frac{1}{2}(100) = 50$. Also, $y = 50$.
26. $\text{arc}(AXB) = 50^\circ$, therefore, $\angle 1 = \angle 2 = 25^\circ$.
27. $\angle 1 = \angle 2$ (base angles are congruent for isosceles triangle), $\angle 2 = \frac{1}{2}\text{arc}(AC)$. $\angle 3 = \frac{1}{2}\text{arc}(AC)$. Thus, $\angle 1 = \angle 3$ (alternate interior angle) $\Rightarrow DE \parallel BC$.

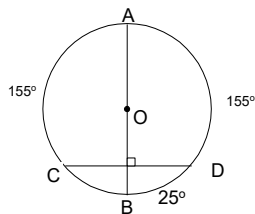


28. $5x = \frac{1}{2}\text{arc}(ADC)$, $4x = \frac{1}{2}\text{arc}(ABC)$. Therefore, $5x + 4x = \frac{1}{2}[\text{arc}(ADC) + \text{arc}(ABC)] = \frac{1}{2}(360^\circ) = 180 \Rightarrow x = 20$.
29. $\text{arc}(BD) = \text{arc}(CE)$, $\angle 1 = \frac{1}{2}(\text{arc CE} + \text{arc BC}) = \frac{1}{2}(\text{arc BD} + \text{arc BC}) = \angle 2$. $\Rightarrow \angle 1 = \angle 2$ and $\triangle ADE$ is an isosceles triangle.
30. $\triangle ABC$ is an isosceles since $\angle 3 = \angle 4$ for $\angle 3 = \angle 1$ and $\angle 4 = \angle 2$.
31. $\angle ACB = \frac{1}{2}\text{arc}(ADB) = \frac{1}{2}(180^\circ) = 90^\circ$. Thus, $\triangle ABC$ is a right triangle and the diameter is the hypotenuse.



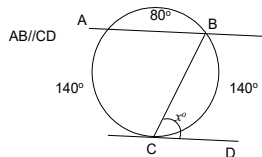
32. $x = 2(60) = 120$
33. $\text{arc}(AC) = 70^\circ$
 $x = \text{arc}(CD) = 360 - 2(70) - 60 = 160$
34. $x = \angle ACB = \frac{1}{2}\text{arc}(ADB) = \frac{1}{2}(180) = 90$. $\triangle ABC$ is a right triangle. Thus, $y = 90 - 37 = 53$. The diameter = hypotenuse $= \sqrt{12^2 + 16^2} = 20$ cm. Therefore, the radius is 10 cm. The area $= 100\pi \text{ cm}^2$

35. 25°

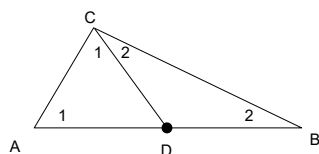


36. 70

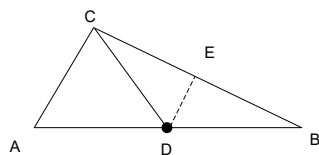
Since $AB \parallel CD$, $\text{arc } BC = \text{arc } AC$. Thus, $\text{arc } BC = 140^\circ$. $\angle BCD = \frac{1}{2}(\text{arc } BC)$. Therefore, $x = \frac{1}{2}(140) = 70$.



37.



38. Draw an auxiliary line DE, parallel to AC, then $BE = CE$ since $AD = BD$. Now that $DE \perp BC$ (corresponding angle to a right angle), DE is a perpendicular bisector to BC, thus $BD = CD (=AD)$.



39. $DA = DC = DB$, therefore, D is the midpoint of AB.

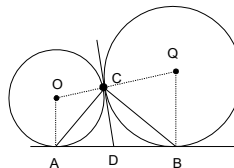
40. Method I:

Since D is equidistant to A, B and C. We can use D as the center draw a circle, AB becomes a diameter, so $\angle ACB = 90^\circ$

Method II:

Let O and Q be the centers of the two circles.

AOQB is a right trapezoid with $AO \parallel BQ$. $\angle CQB + \angle AOC = 180^\circ$, $\angle DCA = \frac{1}{2}\angle AOC$, $\angle BCD = \frac{1}{2}\angle CQB$, $\angle ACB = \angle ACD + \angle DCB = \frac{1}{2}(\angle AOC + \angle CQB) = \frac{1}{2}(180^\circ) = 90^\circ$.



41. Right trapezoid

42. $OQ = 13$, $AB^2 = OQ^2 - (OA - QB)^2 = 13^2 - (8 - 3)^2 = 12^2$, $AB = 12$

43. $\frac{1}{2}(3+8)(12) = 66$

44. Let x be the radius of circle O. We have

$$(x+6)^2 - (x-3)^2 = 15^2$$

$$x^2 + 12x + 36 - x^2 + 6x - 9 = 225$$

$$18x = 198$$

$$x = 11$$

45. $\frac{1}{2}(3+8)(12) = 66$

46. 5

$OQ = 12 + r - 4$, since $AO^2 + AQ^2 = OQ^2$, we have

$$12^2 + r^2 = (8 + r)^2$$

$$144 + r^2 = 64 + 16r + r^2$$

$$16r = 80$$

$$r = 5$$