CALCULUS BC

SECTION I, Part A

Time—60 Minutes

Number of questions—30

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. Evaluate $\int_1^{\infty} 4xe^{-x^2}\,dx$.

(A) $\frac{2}{e^2}$

(B) $\frac{2}{e}$

(C) 2

(D) The integral diverges.

2. The table below gives some values of a function f and its first three derivatives. What is the third-degree Taylor Polynomial for f about $x = 1$?

x	$f(x)$	$f'(x)$	$f''(x)$	$f'''(x)$
0	5	3	1	2
1	1	2	4	18
2	3	7	16	32

(A) $3x^3 + 7x^2 + 16x + 32$

(B) $3x^3 + 7x^2 + 7x + 2$

(C) $3x^3 - 7x^2 + 7x - 2$

(D) $3x^3 - 7x^2 + 16x - 32$

$$f(x) = f(1) + f'(1)(x-1) + \frac{f''(1)}{2!}(x-1)^2 + \frac{f''(1)}{3!}(x-1)^3 + \cdots$$

$$3(x-1)^3 + 2(x-1)^2 + 2(x-1) + 1$$
$$= 3x^3 - 7x^2 + 7x - 2$$

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3. Which of the following is true about the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$?

(A) The series converges conditionally.

(B) The series diverges.

(C) The series converges absolutely.

(D) None of the above

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$
$$e^{-1} = 1 + (-1) + \frac{(-1)^2}{2!} + \frac{(-1)^3}{3!} + \cdots$$
$$e^{-1} - 1 = (-1) + \frac{(-1)^2}{2!} + \frac{(-1)^3}{3!} + \cdots$$

GO ON TO THE NEXT PAGE.

4. The position of a particle is given by the parametric equations $x(t)=8\sqrt{3t+1}$ and $y(t)=9\sqrt[3]{t^2+2}$. Find the slope of the tangent line to the path of the particle at the time t = 5 seconds.

(A) 30

(B) 10

(C) $\frac{10}{3}$

(D) $\frac{10}{9}$

$$\frac{dy}{dx}\Big|_{t=5} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}\Big|_{t=5}$$

$$= \frac{9\,\frac{1}{3}(t^2+2)^{-\frac{2}{3}}(2t)}{8\,\frac{1}{2}(3t+1)^{-\frac{1}{2}}(3)}\Big|_{t=5}$$

$$= \frac{9}{8}\cdot\frac{\frac{1}{3}\left(27^{-\frac{2}{3}}\right)(10)}{\frac{1}{2}\left(16^{-\frac{1}{2}}\right)(3)}$$

$$= \frac{9}{8}\cdot\frac{\frac{1}{3}\left(\frac{1}{9}\right)(10)}{\frac{1}{2}\left(\frac{1}{4}\right)(3)} = \frac{\frac{10}{3}}{3} = \frac{10}{9}$$

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5. Evaluate $\int \frac{9x-13}{(x-1)(x-3)}dx$.

(A) $9\ln|x-1|+16\ln|x-3|+C$

(B) $2\ln|x-1|+7\ln|x-3|+C$

(C) $7\ln|x-1|+2\ln|x-3|+C$

(D) $9\ln|x-1|-16\ln|x-3|+C$

$$\frac{9x-13}{(x-1)(x-3)} = \frac{a}{x-1} + \frac{b}{x-3}$$

$$9x-13 = a(x-3)+b(x-1)$$

Plug 1 into x: -4 = -2$a \Rightarrow a = 2$

Plug 3 into x: 14 = 2$b \Rightarrow b = 7$

$$\int \frac{9x-13}{(x-1)(x-3)}\,dx = 2\ln(x-1)+7\ln(x-3)+c$$

GO ON TO THE NEXT PAGE.

6. Which of the following equations gives the length of the curve $f(x) = 6\sin(2x)$ from $x = 0$ to $x = 4$?

(A) $\int_0^4 \sqrt{1+36\sin^2(2x)}\,dx$

(B) $\int_0^4 \sqrt{1+144\sin^2(2x)}\,dx$

(C) $\int_0^4 \sqrt{1+36\cos^2(2x)}\,dx$

(D) $\int_0^4 \sqrt{1+144\cos^2(2x)}\,dx$

$f'(x) = 12\cos(2x)$

$\int_0^4 \sqrt{1+f'(x)^2}dx$

$= \int_0^4 \sqrt{1+(12\cos 2x)^2}dx$

$= \int_0^4 \sqrt{1+144\cos^2(2x)}\,dx$

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7. Which of the following is the particular solution to $\frac{dy}{dx} = 3x + e^{2x}$ with the initial condition $y(0) = 5$?

(A) $\frac{3x^2}{2} + \frac{e^{2x}}{2} + \frac{9}{2}$

(B) $\frac{3x^2}{2} + \frac{e^{2x}}{2} + \frac{11}{2}$

(C) $\frac{3x^2}{2} + \frac{e^{2x}}{2} + 3$

(D) $\frac{3x^2}{2} + \frac{e^{2x}}{2} - \frac{9}{2}$

GO ON TO THE NEXT PAGE.

8. Let $y = f(x)$ be the solution of the differential equation $\frac{dy}{dx} = 6y - 2$ with the initial condition $f(0) = 2$. Use Euler's Method to approximate $f(1)$ starting at $x = 0$ with the step size of 0.5.

(A) 7
(B) 10
(C) 27
(D) 40

$y(0.5) = y(0) + y'(0)(0.5) = 2 + (6(2) - 2)(0.5) = 7$
$y(1) = y(0.5) + y'(0.5)(0.5) = 7 + (6(7) - 2)(0.5) = 27$

---

9. Evaluate $\int 3x \sin\left(\frac{x}{3}\right) dx$.

Take the derivative for each answer choice.

(A) $-9x\cos\left(\frac{x}{3}\right) + C$

(B) $-9x\cos\left(\frac{x}{3}\right) + 27\sin\left(\frac{x}{3}\right) + C$

(C) $-9x\cos\left(\frac{x}{3}\right) - 27x\sin\left(\frac{x}{3}\right) + C$

(D) $-x\cos\left(\frac{x}{3}\right) + C$

GO ON TO THE NEXT PAGE.

10. Find the area between the curve $y = 6x - x^2$ and the x-axis.

(A) 324

(B) 180

(C) 72

(D) 36

$\int_0^6 6x - x^2\,dx = 3x^2 - \frac{1}{3}x^3|_0^6 = 108 - 72 = 36$

---

11. Find $\frac{dy}{dx}$ if $y = \cos^4 x$.

(A) $-4\sin^3 x$

(B) $4\sin^3 x$

(C) $-4\cos^3 x \sin x$

(D) $4\cos^3 x \sin x$

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12. If R is the region between the curves $y = x^2 - 4x$ and $y = x + 6$, find the area of R.

(A) $48\frac{1}{6}$

(B) $57\frac{1}{6}$

(C) $57\frac{5}{6}$

(D) $60\frac{1}{6}$

$y_1 - y_2 = x^2 - 5x - 6 = (x-6)(x+1) = 0$

x = -1 or 6

$\int_{-1}^{6} |x^2 - 5x - 6|\ dx$

$= ||\ \frac{1}{3}x^3 - \frac{5}{2}x^2 - 6x|_{-1}^{6}\ ||$

$= |\frac{1}{3}(217) - \frac{5}{2}(35) - 6(7)|$

$= 57\frac{1}{6}$

GO ON TO THE NEXT PAGE.

13. The MacLaurin series for the function f is given by $f(x)=\sum_{n=0}^{\infty}\left(\frac{x}{5}\right)^{n}$. What is the value of $f(2)$?

(A) 0

(B) $\frac{5}{7}$

(C) 1

(D) $\frac{5}{3}$

$$\frac{1}{1-x}=1+x+x^2+x^3+\cdots$$

$$1+\frac{x}{5}+\left(\frac{x}{5}\right)^2+\left(\frac{x}{5}\right)^3+\cdots=\frac{1}{1-\frac{x}{5}}$$

$$1+\frac{2}{5}+\left(\frac{2}{5}\right)^2+\left(\frac{2}{5}\right)^3+\cdots=\frac{1}{1-\frac{2}{5}}=\frac{5}{3}$$

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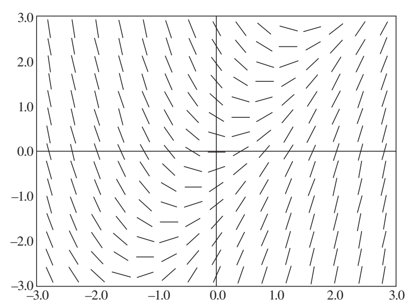
14. Which of the following is the differential equation of the slope field below?



(A) $\frac{dy}{dx}=2x+y$

(B) $\frac{dy}{dx}=2x-y$

(C) $\frac{dy}{dx}=x^2+y$

(D) $\frac{dy}{dx}=x^2-y$

15. A curve is defined by the parametric equations $x = t^4 - t^2 + 1$ and $y = t^3$. Which of the following is the equation of the line tangent to the graph at (13, 8) ?

(A) $7x - 3y = -17$
(B) $3x - 7y = 17$
(C) $7x - 3y = 17$
(D) $3x - 7y = -17$

For the point (13, 8), $t = 2$.

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2}{4t^3-2t}\Big|_{t=2} = \frac{12}{28} = \frac{3}{7}$$

$$y = \frac{3}{7}x + c$$

or

$3x - 7y = c'$ through (13, 8), so $c' = -17$

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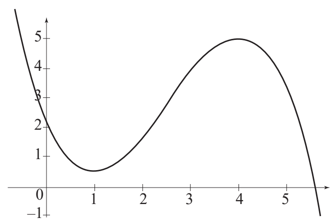
16. The graph of the function f is shown below.



If $g(x) = \int_5^{4x} f(t)\,dt$, what is $g'(1)$?

(A) 0
(B) 3
(C) 20
(D) 27

Fundamental theorem of calculus and chain rule:
If $g(x) = \int_a^u f(t)\ dt$, then $g'(x) = f(u)u'(x)$

So, $g'(x) = f(4x)(4)$
Thus, $g'(1) = f(4) \cdot (4) = 5 \times 4 = 20$

GO ON TO THE NEXT PAGE.

17. The sum of the infinite geometric series $5+\frac{10}{\pi}+\frac{20}{\pi^2}+\frac{40}{\pi^3}+...$ is

(A) $\frac{5\pi}{\pi-2}$

(B) $\frac{5}{\pi-2}$

(C) $\frac{5\pi}{\pi+2}$

(D) $\frac{5}{\pi+2}$

$$5+\frac{10}{\pi}+\frac{20}{\pi^2}+\frac{40}{\pi^3}+\cdots$$
$$=5(1+\frac{2}{\pi}+\frac{4}{\pi^2}+\frac{8}{\pi^3}+\cdots) \text{ (geometric series with } r=\frac{2}{\pi}<1)$$
$$=5(1+\frac{2}{\pi}+\frac{4}{\pi^2}+\frac{8}{\pi^3}+\cdots)$$
$$=5\,\frac{1}{1-\frac{2}{\pi}}$$
$$=\frac{5\pi}{\pi-2}$$

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18. The average value of $\frac{4}{1+x^2}$ on the interval [0, 1] is

(A) π

(B) 2π

(C) 4π

(D) 16π

Avg(*f*) on the interval $[a, b]=\frac{\int_a^b f(x)\,dx}{b-a}$

$$\frac{\int_0^1 \frac{4}{1+x^2}\,dx}{1-0}=4\text{atan}\,x\,|_0^1=\pi$$

GO ON TO THE NEXT PAGE.

19. The function f is given by $f(x) = 9x^4 - 8x^3 - 96x^2 + 10$. On which of the following intervals is f increasing?

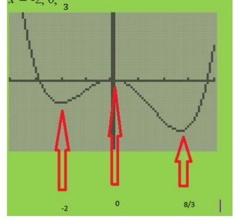
(A) $(-\infty, -2)$

(B) $(0, 2)$

(C) $\left(2, \frac{8}{3}\right)$

(D) $\left(\frac{8}{3}, \infty\right)$

$f'(x) = 36x^3 - 24x^2 - 192x$
$= 2x(3x^2 - 2x - 16)$
$= 2x(3x - 8)(x + 2) = 0$
$x = -2, 0, \frac{8}{3}$



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20. What are all values of x for which $\sum_{k=0}^{\infty} \frac{(x-1)^k}{k(4^k)}$ converges?

(A) $-3 < x < 5$

(B) $-3 \le x \le 5$

(C) $-3 \le x < 5$

(D) The series diverges.

Root test $\sqrt[k]{\frac{(x-1)^k}{k(4^k)}} = \frac{|x-1|}{\sqrt[k]{k}4}$

Note: $\lim_{k\to\infty} \sqrt[k]{k} = \lim_{k\to\infty} e^{\frac{\ln k}{k}} = e^{\lim_{k\to\infty} \frac{\ln k}{k}} = e^0 = 1$

Therefore, $\frac{|x-1|}{\sqrt[k]{k}4} = \frac{|x-1|}{4} < 1$

$|x - 1| < 4$

the interval of convergence is $-3 < x < 5$.

On the boundaries,

when $x = 5$, $\sum_{k=0}^{\infty} \frac{1}{k}$ is divergent;

when $x = -3$, $\sum_{k=0}^{\infty} \frac{(-1)^k}{k}$ converges, so the interval of convergence is $-3 \le x < 5$.

GO ON TO THE NEXT PAGE.

Section I

21. Find the area between the polar curves $r = \cos\theta$ and $r = 2\cos\theta$ from $\theta = 0$ to $\theta = \pi$.

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{2}$

(C) $\frac{3\pi}{4}$

(D) $\frac{3\pi}{2}$

Method I)
The area between two circles with radii $\frac{1}{2}$ and 1 is $\frac{3}{4}\pi$.

Method II)

$$\frac{1}{2}\int_0^{\pi} 4\cos^2\theta - \cos^2\theta \, d\theta$$
$$= \frac{1}{2}\int_0^{\pi} 3\cos^2\theta \, d\theta$$
$$= \frac{3}{2}\int_0^{\pi} \cos^2\theta \, d\theta$$

$$\cos^2\theta = \frac{1}{2}(\cos 2\theta + 1)$$

$$\frac{3}{2}\int_0^{\pi} \cos^2\theta \, d\theta$$
$$= \frac{3}{2}\int_0^{\pi} \frac{1}{2}(\cos 2\theta + 1) \, d\theta$$
$$= \frac{3}{4}\int_0^{\pi} \cos 2\theta + 1 \, d\theta$$
$$= \frac{3\pi}{4}$$

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22. Evaluate $\int_3^4 \frac{dx}{(x-3)^2}$.

(A) –1
(B) 0
(C) ln 4
(D) Divergent

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23. A car is accelerating at 20 meters per second squared from an initial velocity of 10 meters per second. How far does the car travel in the first six seconds?

(A) 120 m
(B) 130 m
(C) 420 m
(D) 780 m

GO ON TO THE NEXT PAGE.

24. The value of c that satisfies the Mean Value Theorem for Derivatives on the interval [3, 4] for the function $f(x) = x + \frac{1}{x}$ is

(A) $-\sqrt{12}$

(B) -1

(C) 1

(D) $\sqrt{12}$

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25. For $x > 0$, the power series $x + x^2 + \frac{x^3}{2!} + \frac{x^4}{3!} + \ldots + \frac{x^n}{(n-1)!} + \ldots$ converges to which of the following?

(A) xe^x

(B) $\frac{e^x}{x}$

(C) e^{x^2}

(D) The series does not converge.

GO ON TO THE NEXT PAGE.

26. If $f(x) = x^2 \ln x$, for what value of x is the tangent line horizontal?

(A) $-\frac{1}{2}$

(B) $e^{-\frac{1}{2}}$

(C) 1

(D) $e^{\frac{1}{2}}$

27. Given $x^3 + 2xy = y^3 + x + 9$, find $\frac{d^2y}{dx^2}$ at (2, 1).

(A) –1,054
(B) –13
(C) 13
(D) 1,054

The curve is defined by $f(x,y) = x^3 - y^3 + 2xy - x - 9 = 0$

$f_x dx + f_y dy = 0$

$\frac{dy}{dx} = \frac{-f_x}{f_y} = -\frac{3x^2+2y-1}{-3y^2+2x} = \frac{3x^2+2y-1}{3y^2-2x}$

$\frac{dy}{dx}(2,1) = \frac{12+2-1}{-1} = -13$

$\frac{d^2y}{dx^2} = \frac{d}{dx}\frac{dy}{dx} = \frac{(6x+2y')(3y^2-2x)-(3x^2+2y-1)(6yy'-2)}{(3y^2-2x)^2}$

$\frac{d^2y}{dx^2}(2,1) = \frac{(12-26)(-1)-(13)(6\cdot(-13)-2)}{1^2} = 1054$

28. If $\int_0^4 f(x)dx = 20$ and $\int_4^2 f(x)dx = 11$, then $\int_0^2 f(x)dx =$

(A) –31
(B) –9
(C) 9
(D) 31

GO ON TO THE NEXT PAGE.

29. Evaluate $\lim_{h \to 0} \frac{\ln(4+h) - \ln 4}{h}$.

(A) 0

(B) $\frac{1}{4}$

(C) 4

(D) The limit does not exist.

30. Evaluate $\int_1^{\infty} \frac{dx}{\sqrt[3]{x^5}}$.

(A) $-\frac{3}{2}$

(B) 0

(C) 1

(D) $\frac{3}{2}$

END OF PART A, SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS BC

SECTION I, Part B

Time—45 Minutes

Number of questions—15

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test:

1. The **exact** numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

2. Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

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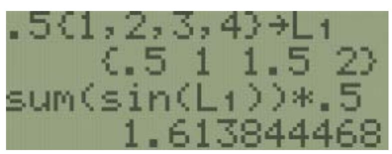
31. Approximate the area under $y = \sin x$ from $x = 0$ to $x = 2$, using $n = 4$ right endpoint rectangles.

(A) 0.044
(B) 1.614
(C) 2.318
(D) 4.636



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32. The radius of a circle is increasing at 1.5 centimeters per second. How fast is the area increasing when $r = 4$ cm?

(A) 7.069
(B) 25.133
(C) 37.699
(D) 75.398

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}\Big|_{r=4} = 2\pi(4)(1.5) = 12\pi$

12π
37.69911184

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33. For which of the following values of x is the slope of the tangent line to $y_1 = 2\sin x$ equal to the slope of the tangent line to $y_2 = \tan x$ on the interval $[0, 2\pi]$?

(A) 0
(B) 0.654
(C) 1.047
(D) There is no value of x.

$y_1' = 2\cos x$

$y_2' = \sec^2 x$

$2\cos x = \sec^2 x$

$\cos^3 x = \frac{1}{2}$

$\cos x = \sqrt[3]{\frac{1}{2}}$

cos⁻¹(1/2^(1/3))
.6539279425

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34. Using the Taylor series about $x = 0$ for e^x, approximate $e^{0.2}$ to three decimal places.

(A) 1.221
(B) 1.249
(C) 1.250
(D) 7.389

$e^{.2}$
1.221402758

GO ON TO THE NEXT PAGE.

35. What is the maximum value of $y = \frac{\ln x}{x}$ on the interval [1, 5] ?

(A) 0
(B) 0.322
(C) 0.368
(D) 2.718

$$y' = \frac{1-\ln x}{x^2} = 0$$
$$x = e$$
$$y(e) = \frac{1}{e}$$

e^{-1}

.3678794412

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36. If $f(x) = \frac{d}{dx}\int_4^{6x} \sin t\, dt$, find $f(0.1)$.

(A) 0.099
(B) 0.565
(C) 3.388
(D) 6.600

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37. Evaluate $\int \sin^3 x \cos^2 x\, dx$.

(A) $\frac{\cos^3 x}{3} - \frac{\cos^5 x}{5} + C$

(B) $-\frac{\cos^3 x}{3} + \frac{\cos^5 x}{5} + C$

(C) $-\frac{\sin^3 x}{3} + \frac{\sin^5 x}{5} + C$

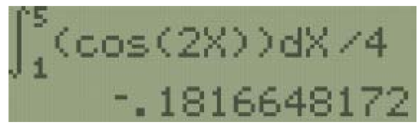
(D) $\frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + C$

$$\int \sin^2 x \cos^2 x\, (\sin x\, dx)$$
$$= -\int \sin^2 x \cos^2 x\, d\cos x$$
$$= -\int (1-\cos^2 x)\cos^2 x\, d\cos x$$
$$= -\int (1-u^2)u^2 du \text{ (Let } u = \cos x\text{)}$$
$$= -\int u^2 - u^4 du$$
$$= -\frac{1}{3}u^3 + \frac{1}{5}u^5 + c$$
$$= -\frac{1}{3}\cos^3 x + \frac{1}{5}\cos^5 x + c$$

GO ON TO THE NEXT PAGE.

38. What is the average value of $y = \cos 2x$ on the interval [1, 5] ?

(A) -1.453
(B) -0.182
(C) 0.182
(D) 1.453



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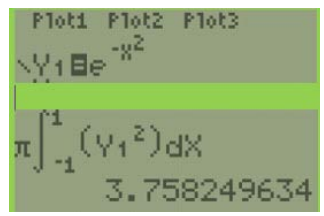
39. Let R be the region between $y = e^{-x^2}$ and the x-axis from $x = -1$ to $x = 1$. Find the volume of the solid that results when R is revolved about the x-axis.

(A) 1.196
(B) 3.758
(C) 7.516
(D) 14.124



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40. Evaluate $\int e^{2x} \cos 2x \, dx$.

(A) $e^{2x}\sin 2x - e^{2x}\cos 2x + C$

(B) $e^{2x}\sin 2x + e^{2x}\cos 2x + C$

(C) $\frac{1}{4}e^{2x}\sin 2x - \frac{1}{4}e^{2x}\cos 2x + C$

(D) $\frac{1}{4}e^{2x}\sin 2x + \frac{1}{4}e^{2x}\cos 2x + C$

$\frac{1}{2}\int e^{2x}\cos 2x \; d2x = \frac{1}{2}\int e^{u}\cos u \; du$

$\int e^{u}\cos u \; du = \cos u \cdot e^{u} + \int e^{u}\sin u \, du \; \ldots$ ①

$\int e^{u}\sin u \, du = \sin u \cdot e^{u} - \int e^{u}\cos u \, du \ldots$ ②

① + ②

$2\int e^{u}\cos u \; du = e^{u}(\cos u + \sin u)$

$\int e^{u}\cos u \; du = \frac{1}{2}e^{u}(\cos u + \sin u) + c$

$\int e^{2x}\cos 2x \; dx$

$= \frac{1}{2}\int e^{u}\cos u \; du$

$= \frac{1}{4}e^{2x}(\cos 2x + \sin 2x) + c$

GO ON TO THE NEXT PAGE.

41. Evaluate $\lim_{x \to 0} \frac{x - \sin x}{e^x - 1}$.

(A) 0

(B) 1

(C) $\frac{1}{e}$

(D) The limit does not exist.

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42. The equation of the line normal to the graph of $y = 6^{x^2}$ at $x = 1$ is

(A) $y - 6 = \frac{1}{12 \ln 6}(x - 1)$

(B) $y - 6 = -\frac{1}{12 \ln 6}(x - 1)$

(C) $y - 6 = 12 \ln 6 (x - 1)$

(D) $y - 6 = -12 \ln 6 (x - 1)$

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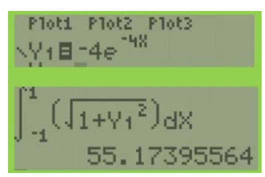
43. What is the length of the curve $y = e^{-4x}$ from $x = -1$ to $x = 1$?

(A) 1.056
(B) 1.111
(C) 4.505
(D) 55.174



GO ON TO THE NEXT PAGE.

44. Evaluate $\int \frac{dx}{\sqrt{36-x^2}}$.

(A) $\sin^{-1} 6x + C$

(B) $\sin^{-1} \frac{x}{6} + C$

(C) $\frac{1}{2x}\sqrt{36-x^2} + C$

(D) $-\frac{1}{2x}\sqrt{36-x^2} + C$

$$\int \frac{dx/6}{\sqrt{36-x^2/6}} = \int \frac{d\frac{x}{6}}{\sqrt{1-\left(\frac{x}{6}\right)^2}} = \text{asin}\frac{x}{6} + c$$

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45. Evaluate $\int x^3\sqrt{x^2-1}\,dx$.

(A) $\frac{(x^2-1)^{\frac{3}{2}}}{3} + \frac{(x^2-1)^{\frac{5}{2}}}{5} + C$

(B) $\frac{x^4}{4}\left(\frac{2}{3}(x^2-1)^{\frac{3}{2}}\right) + C$

(C) $3x^2\left(-\frac{1}{2}(x^2-1)^{-\frac{1}{2}}\right) + C$

(D) $\frac{x^4}{4} + \frac{2}{3}(x^2-1)^{\frac{3}{2}} + C$

$$\int x^2\sqrt{x^2-1}\; xdx$$
$$= \int x^2\sqrt{x^2-1}\;\left(\frac{1}{2}dx^2\right)$$
$$= \frac{1}{2}\int x^2\sqrt{x^2-1}\;dx^2 \quad (\text{Let } u = x^2-1, x^2 = u+1)$$
$$= \frac{1}{2}\int (u+1)\sqrt{u}\;du$$
$$= \frac{1}{2}\int u^{\frac{3}{2}} + u^{\frac{1}{2}}\;du$$
$$= \frac{1}{5}u^{\frac{5}{2}} + \frac{1}{3}u^{\frac{3}{2}} + c$$
$$= \frac{1}{5}(x^2-1)^{\frac{5}{2}} + \frac{1}{3}(x^2-1)^{\frac{3}{2}} + c$$

STOP

END OF PART B, SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART B ONLY.

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.

SECTION II, PART A
Time—30 minutes
Number of problems—2

A graphing calculator is required for some problems or parts of problems.

During the timed portion for Part A, you may work only on the problems in Part A.

On Part A, you are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

1. Grain is being loaded into a silo at the rate of $G(t) = 400e^{\frac{-t^2}{4}}$ cubic feet per hour, where t is the number of hours that it is being loaded, $0 \le t \le 8$. At time $t = 0$, there is 100 cubic feet of grain in the silo. Grain is also being removed through the base of the silo at the following rates, where $R(t)$ is the amount of grain being removed in cubic feet per hour, $0 \le t \le 8$:

t	0	2	5	7	8
$R(t)$	60	90	110	120	125

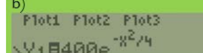
(a) Estimate the total amount of grain removed from the silo at $t = 8$ hrs, using a left-hand Riemann sum and 4 subintervals.

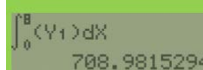
(b) Estimate the amount of grain in the silo at the end of 8 hours, using your answer from part (a).

(c) Estimate $R'(5)$, showing your work. Indicate the units of measure.

a) $R(0)(2-0)+R(2)(5-2)+R(5)(7-5)+R(7)(8-7)$
$= 60\times2 + 90\times3 + 110\times2 + 120\times1$
$= 730$

b)





c) $R'(5^-) = \frac{R(5)-R(2)}{3} = \frac{20}{3}$
$R'(5^+) = \frac{R(7)-R(5)}{2} = 5$
$R'(5) = \frac{6.67+5}{2} = 5.83$

2. Consider the function given by $f(x) = x^2 e^{-4x}$.

(a) Find $\lim_{x\to\infty} f(x)$.

(b) Find the maximum value of f on the interval $[0, \infty]$. Justify your answer.

(c) Evaluate $\int_0^\infty f(x)\,dx$, or show that the integral diverges.

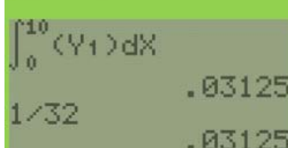
a) $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x^2}{e^{4x}} = \lim_{x\to\infty} \frac{2x}{4e^{4x}} = \lim_{x\to\infty} \frac{2}{16e^{4x}} = 0$ (L'Hopital's rule)

b) $f'(x) = 2x(1-2x)e^{-4x} = 0$
$x = 0$ or ½

$\max_{0\le x\le\infty} f(x) = f\left(\frac{1}{2}\right) = \frac{1}{4e^2}$

c) $\int x^2e^{-4x}dx = \left(-\frac{1}{4}x^2 + ax + b\right)e^{-4x} + c$
Derive on both sides:
$x^2e^{-4x} = \left(x^2 - \left(\frac{1}{2} + 4a\right)x + (a - 4b)\right)e^{-4x}$
$a = -\frac{1}{8}$
$b = \frac{1}{32}$
$\int_0^\infty x^2e^{-4x}dx = \left(-\frac{1}{4}x^2 - \frac{1}{8}x + \frac{1}{32}\right)e^{-4x}\Big|_0^\infty$
$= \frac{1}{32}$

Verify:



SECTION II, PART B
Time—1 hour
Number of problems—4

No calculator is allowed for these problems.

During the timed portion for Part B, you may continue to work on the problems in Part A without the use of any calculator.

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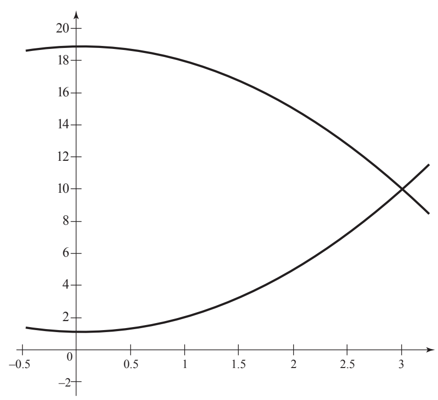
3. Let R be the region in the first quadrant bounded from above by $g(x) = 19 - x^2$ and from below by $f(x) = x^2 + 1$.



(a) Find the area of R.

(b) A solid is formed by revolving R around the x-axis. Find the volume of the solid.

(c) A solid has its base as the region R, whose cross-sections perpendicular to the x-axis are squares. Find the volume of the solid.

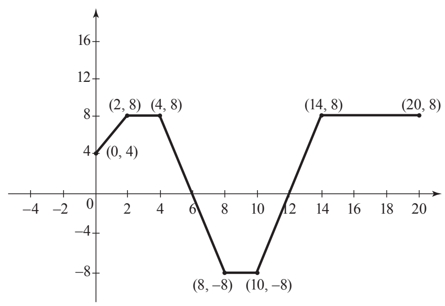
a) $\int_0^3 g(x) - f(x)\ dx$
$= \int_0^3 18 - 2x^2\ dx$
$= 18x - \frac{2}{3}x^3 |_0^3$
$= 54 - 18$
$= 36$

b) $\pi \int_0^3 g^2(x) - f^2(x)\ dx$
$= \pi \int_0^3 (x^4 - 38x^2 + 361) - (x^4 + 2x^2 + 1)\, dx$
$= \pi \int_0^3 -36x^2 + 360\ dx$
$= \pi(360x - 12x^3)|_0^3$
$= \pi(1080 - 324)$
$= 756\pi$

c) $\pi \int_0^3 \left(g(x) - f(x)\right)^2\ dx$
$= \pi \int_0^3 2^2(x^2 - 9)^2\, dx$
$= 4\pi \int_0^3 x^4 - 18x^2 + 81\, dx$
$= \pi \left(\frac{1}{5}x^5 - 6x^3 + 81x\right) |_0^3$
$= \pi \left(\frac{243}{5} - 162 + 243\right)$
$= 81\pi \left(\frac{2}{5}\right) = 32.4\pi$

GO ON TO THE NEXT PAGE.

4. A particle begins on the y-axis at the point (0, 4) at time $t = 0$, and travels along a straight line. For $0 \le t \le 20$, the particle's velocity, in feet per second, can be modeled by the piecewise-linear function in the graph below.



(a) At what times in the interval $0 \le t \le 20$ does the particle change direction? Explain your answer.

(b) Find the total displacement of the particle in the interval $0 \le t \le 20$.

(c) (i) Write an expression for the particle's velocity, $v(t)$, in the time interval $10 < t < 14$.

(ii) Write an expression for the particle's acceleration, $a(t)$, in the time interval $10 < t < 14$.

(d) Write an expression for the particle's position, $s(t)$, in the time interval $10 < t < 14$.

5. Consider the curve $y^2 - 3xy = -2$.

(a) Find an equation of the tangent line to the curve at the point (1, 1).

(b) Find all x-coordinates where the slope of the tangent line to the curve is undefined.

(c) Evaluate $\frac{d^2y}{dx^2}$ at the point (1, 1).

GO ON TO THE NEXT PAGE.

6. The function f has derivatives of all orders and the MacLaurin series for f is $\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{2n+2} = \frac{1}{2} - \frac{x^2}{4} + \frac{x^4}{6} - \ldots.$

(a) Using the Ratio Test, determine the interval of convergence for the MacLaurin series for f.

(b) The MacLaurin series for f evaluated at $x = \frac{1}{2}$ is an alternating series whose terms decrease in absolute value to 0. The approximation for $f\left(\frac{1}{2}\right)$ using the first three nonzero terms of this series is $\frac{43}{96}$. Show that this approximation differs from $f\left(\frac{1}{2}\right)$ by less than $\frac{1}{1{,}000}$.

(c) Write the first three nonzero terms and the general term of the MacLaurin series for $f'(x)$.

STOP

END OF EXAM