CALCULUS BC

SECTION I, Part A

Time—60 Minutes

Number of questions—30

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. If $y = \tan(2\pi x)$, then $\frac{dy}{dx} =$

(A) $(2\pi)\cot(2\pi x)$

(B) $(2\pi)\sec^2(2\pi x)$

(C) $\frac{\sec^2(2\pi x)}{2\pi}$

(D) $\frac{\cot(2\pi x)}{2\pi}$

9. $\frac{d}{dx}(\sin x) = \cos x$
10. $\frac{d}{dx}(\csc x) = -\csc x \cot x$
11. $\frac{d}{dx}(\cos x) = -\sin x$
12. $\frac{d}{dx}(\sec x) = \sec x \tan x$
13. $\frac{d}{dx}(\tan x) = \sec^2 x$
14. $\frac{d}{dx}(\cot x) = -\csc^2 x$

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2. $\lim_{x \to 0} \frac{\sin x - e^x + 1}{x} =$

(A) 0
(B) 1
(C) −1
(D) The limit does not exist.

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9. $\frac{d}{dx}(\sin x) = \cos x$ 10. $\frac{d}{dx}(\csc x) = -\csc x \cot x$

11. $\frac{d}{dx}(\cos x) = -\sin x$ 12. $\frac{d}{dx}(\sec x) = \sec x \tan x$

13. $\frac{d}{dx}(\tan x) = \sec^2 x$ 14. $\frac{d}{dx}(\cot x) = -\csc^2 x$

3. $\int \tan^2 x \, dx =$

(A) $\tan x - x + C$

(B) $\frac{\tan^3 x}{3} + C$

(C) $\tan x + C$

(D) $\tan x + x + C$

$\tan^2 x = \sec^2 x - 1$
$\int \sec^2 x - 1 \, dx = \tan x - x + c$

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4. For $0 \le t \le \frac{11\pi}{6}$, an object travels along the ellipse given by $x = 2\sin t$, $y = -3\cos t$. At $t = \frac{11\pi}{6}$, the object leaves the ellipse and travels along the line tangent to the path at that point. What is the slope of the tangent line at $t = \frac{11\pi}{6}$?

(A) $\frac{2\sqrt{3}}{3}$

(B) $\frac{-3}{2\sqrt{3}}$

(C) $\frac{-3\sqrt{3}}{2}$

(D) $\frac{-2\sqrt{3}}{3}$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3\sin t}{2\cos t}\Big|_{\frac{11\pi}{6}} = \frac{-3}{2\cdot\sqrt{3}}$$

GO ON TO THE NEXT PAGE.

5. If $f(x)=\begin{cases} x^3-4x, & x\ge 2 \\ 4x-2x^2, & x<2 \end{cases}$, which of the following is true?

(A) $f(x)$ is continuous and differentiable at $x = 2$.
(B) $f(x)$ is differentiable but not continuous at $x = 2$.
(C) $f(x)$ is continuous but not differentiable at $x = 2$.
(D) $f(x)$ is neither continuous nor differentiable at $x = 2$.

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6. $\int \tan^2 x \sec^2 x \, dx =$

(A) $\dfrac{\tan^3 x \sec^3 x}{3} + C$

(B) $\tan^2 x + C$

(C) $\dfrac{\sec^3 x}{3} + C$

(D) $\dfrac{\tan^3 x}{3} + C$

$$d \tan x = \sec^2 x \; dx$$
$$\int \tan^2 x \, d \tan x = \frac{1}{3}\tan^3 x + c$$

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7. The position of a particle at time $t \geq 0$, in seconds, is given by $x = 2t^3 + 3t^2 - 12t$ and $y = 2t^3 - 15t^2 + 24t$. At what value(s) of t is the particle at rest?

(A) $t = 1$
(B) $t = 1, 2, 4$
(C) $t = 2$
(D) $t = 2, -3$

$$x'(t) = 6t^2 + 6t - 12 = 6(t+2)(t-1)$$
$$y'^{(t)} = 6t^2 - 30t + 24 = 6(t-4)(t-1)$$

---

8. $\int 3x \csc^2 x^2 \, dx =$

(A) $\frac{3}{2}\sec^2(x^2) + C$

(B) $\frac{\csc x^3}{x} + C$

(C) $-\frac{3}{2}\cot(x^2) + C$

(D) $3\sec^2(x^2) + C$

$$dx^2 = 2xdx$$
$$xdx = \frac{1}{2}dx^2$$
$$\int \frac{3}{2}\csc^2 x^2 \, dx^2$$
$$= -\frac{3}{2}\cot x^2 + c$$

GO ON TO THE NEXT PAGE.

T3

9. If $f(x) = \log_5(x^3 - x - 1)$, then $f'(2) =$

(A) $\frac{11}{5 \ln 5}$

(B) 11

(C) e^5

(D) $\frac{11}{5}$

$$f(x) = \log_5(x^3 - x - 1) = \frac{\ln(x^3 - x - 1)}{\ln 5}$$

$$f'(5) = \frac{1}{\ln 5}\frac{3x^2 - 1}{x^3 - x - 1}\Big|_2 = \frac{1}{\ln 5} \cdot \frac{11}{5}$$

---

10. What is the value of $\sum_{n=1}^{\infty} \frac{5^{n+1}}{4^n}$?

(A) 1

(B) $\frac{2}{3}$

(C) $\frac{1}{3}$

(D) Diverges

GO ON TO THE NEXT PAGE.

11. If the Maclaurin series for e^x is $\sum_{n=0}^{\infty} \frac{x^n}{n!}$, what is a power series expansion for e^{x^3}?

(A) $1 + \frac{x^3}{3!} + \frac{x^6}{6!} + \frac{x^9}{9!} + \ldots$

(B) $1 + x^3 + \frac{x^6}{2!} + \frac{x^9}{3!} + \ldots$

(C) $x^3 + \frac{x^6}{3!} + \frac{x^9}{6!} + \ldots$

(D) $1 - \frac{x^3}{3!} + \frac{x^6}{6!} - \frac{x^9}{9!} + \ldots$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$$

Replace x by x^3

$$e^x = 1 + x^3 + \frac{x^6}{2!} + \frac{x^9}{3!}$$

---

12. The rate of growth of algae, A, in a pond with respect to time, t, is directly proportional to one-half the amount of algae in the pond. Which differential equation describes this relationship?

(A) $A(t) = \frac{1}{2}kA$

(B) $\frac{dA}{dt} = \frac{1}{2}kA$

(C) $\frac{dA}{dt} = \frac{1}{2}k\frac{A}{t}$

(D) $\frac{dA}{dt} = \frac{1}{2}kAt$

---

13. If $\int_0^{10} f(x)\,dx = 100$ and $\int_{10}^{6} f(x)\,dx = 75$, find $\int_0^{6} f(x)\,dx$.

(A) 175
(B) 25
(C) –25
(D) –175

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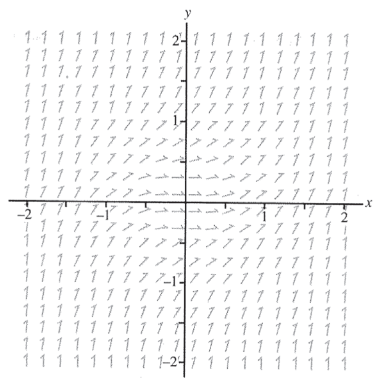

14. Which of the following is the differential equation of the slope field above?

(A) $\frac{dy}{dx} = x^2 + y^2$

(B) $\frac{dy}{dx} = x^2 - y^2$

(C) $\frac{dy}{dx} = x - y$

(D) $\frac{dy}{dx} = (x + y)^2$

GO ON TO THE NEXT PAGE.

15. The length of a curve from $x = 2$ to $x = 3$ is given by $\int_2^3 \sqrt{1+25x^6}\,dx$. If the curve contains the point (2, 10), which of the following could be the curve?

(A) $y = \frac{5x^4}{4}$

(B) $y = \frac{5x^4}{4} - 10$

(C) $y = 25x^2 - 90$

(D) $y = 25x^3 + 10$

Length for the curve defined by $y = f(x)$: $\int \sqrt{1+(f')^2}\,dx$

$y' = 5x^3$

$y = \frac{5}{4}x^4 + c$

---

16. If the line tangent to the graph of the function f at the point (8, 2) passes through the point (2, 5), then $f'(8) =$

(A) -2

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) 2

$$f'(8) = \frac{2-5}{8-2} = \frac{-3}{6} = -\frac{1}{2}$$

GO ON TO THE NEXT PAGE.

17. A curve is defined by the parametric equations $x = t^3 + 2t - 1$ and $y = t^2$. Which of the following is an equation of the line tangent to the graph at (11, 4) ?

(A) $2y - 7x = 69$
(B) $7y - 2x = 6$
(C) $2x - 7y = 6$
(D) $7x + 2y = 69$

$y = t^2 = 4$

$t = \pm 2$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2t}{3t^2+2}\Big|_{\pm 2} = \pm\frac{4}{14} = \pm\frac{2}{7}$

$y = \pm\frac{2}{7}x + c$

$7y \pm 2x = c$

Only B and D can be correct.
So, plugin (11, 4)
Only B works.

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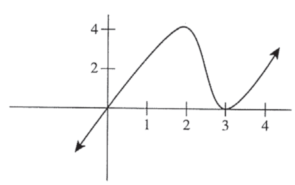
18. The graph of the function f is shown below.



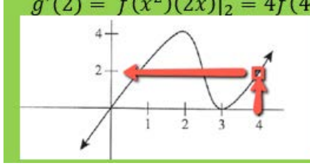
If $g(x) = \int_0^{x^2} f(t)\,dt$, what is $g'(2)$?

(A) 32
(B) 16
(C) 8
(D) 4

$g(x) = \int_0^{u(x)} f(t)\,dt$

$g'(x) = f(u(x))u'(x)$

$g'(2) = f(x^2)(2x)|_2 = 4f(4) = 4 \times 2 = 8$



T3

19. If the graph of $y = f(x)$ has slope $3x^2 + 5$ at each point (x, y) on the curve and it passes through $(2, 18)$, which of the following is the equation of $f(x)$?

(A) $y = 17x - 16$
→ (B) $y = x^3 + 5x$
(C) $y = x^3 + 5x - 1$
(D) $y = 17x - 52$

20. Which of the following are the first four nonzero terms of the Maclaurin series for $f(x) = \sin(x^2)$?

(A) $x^2 - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!}$

(B) $x^2 - \frac{x^6}{6!} + \frac{x^{10}}{10!} - \frac{x^{14}}{14!}$

(C) $x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \frac{x^{14}}{7!}$

(D) $x^2 + \frac{x^6}{3!} + \frac{x^{10}}{5!} + \frac{x^{14}}{7!}$

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} +$

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$

$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$

$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \cdots$

$\frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 - \cdots$

$\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$

$\ln(1 - x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} + \cdots$

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21. The number of new members of a social media site is modeled by the function y that satisfies the differential equation $\frac{dy}{dt} = 25y(1 - \frac{y}{10{,}000})$, where t is the time in days and $y(0) = 100$. What is $\lim_{t\to\infty} y(t)$?

(A) 100
(B) 2500
(C) 10,000
(D) 100,000

22. For what values of x does the series $\sum_{n=1}^{\infty} \frac{5^n}{n^2} x^n$ converge?

(A) $-\frac{1}{5} \le x \le \frac{1}{5}$

(B) $x \ge \frac{1}{5}$

(C) $-5 \le x \le 5$

(D) All real numbers

Section I

T3

23. $\int xe^{6x}\,dx =$

(A) $6xe^{6x} - 36e^{6x} + C$

(B) $\frac{x}{6}e^{6x} - \frac{1}{36}e^{6x} + C$

(C) $\frac{x}{6}e^{6x} + C$

(D) $\frac{x}{36}e^{6x} + C$

$\int ue^u\,du = ue^u - e^u + c$

$\int xe^{6x}\,dx$
$= \frac{1}{36}\int 6xe^{6x}\,d6x$
$= \frac{1}{36}\int ue^u\,du$
$= \frac{1}{36}ue^u - \frac{1}{36}e^u + c$

By letting $u = 6x$
$\int xe^{6x}\,dx = \frac{1}{6}xe^{6x} - \frac{1}{36}e^{6x} + c$

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24. Which of the following series converge(s)?

(I) $\sum_{n=1}^{\infty}\left(\frac{5}{\pi}\right)^n$

(II) $\sum_{n=1}^{\infty} n^{-\frac{3}{2}}$

(III) $\sum_{n=1}^{\infty} \frac{n!}{n^2}$

(A) I only
(B) II only
(C) I and II
(D) II and III

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25. The function f is continuous on the closed interval [0, 10] and has the values in the following table:

x	0	2	5	7	10
$f(x)$	8	11	20	29	34

Using the subintervals [0, 2], [2, 5], [5, 7], and [7, 10], approximate $\int_0^{10} f(x)\,dx$ using a left Riemann sum.

(A) 102
(B) 176
(C) 204
(D) 242

8×(2 – 0) + 11×(5 – 2) + 20×(7 – 5) + 29×(10 – 7)
= 16 + 33 + 40 + 87
= 176

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26. $\int \frac{x+8}{(x-2)(x+3)}\,dx =$

(A) $2\ln|x-2| - \ln|x+3| + C$

(B) $2\ln|x-2| + \ln|x+3| + C$

(C) $\ln|x^2 + x - 6| - 8 + C$

(D) $\ln|x^2 + x - 6| + 8 + C$

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27. Evaluate $\int_{10}^{\infty} \frac{e^{-x}\,dx}{1-e^{-x}}$.

(A) 0

(B) $\ln\left|e^{-10}\right|$

(C) $-\ln\left|1-e^{-10}\right|$

(D) Diverges

Positive answer,
B is negative
C is the only one.

Or,

$$\int \frac{e^{-x}}{1-e^{-x}}dx = \int \frac{-de^{-x}}{1-e^{-x}} = \int \frac{d(1-e^{-x})}{1-e^{-x}} = \ln(1-e^{-x})$$

$$\ln(1-e^{-x})\,|_{10}^{\infty} = -\ln(1-e^{-10})$$

---

28. What is the coefficient of x^8 in the Taylor series for e^{x^2} about $x = 0$?

(A) $\frac{1}{24}$

(B) $\frac{1}{6}$

(C) 6

(D) 24

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} +$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \cdots$$

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 - \cdots$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$$

$$\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} + \cdots$$

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29. If $P(x) = x - 2x^2 + 3x^4 - 4x^6$ is the sixth-degree Taylor polynomial for the function f about $x = 0$, what is $f''(0)$?

(A) −120
(B) −4
(C) 1
(D) 72

30.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	2	−6	−3	−4
1	0	−1	4	6
2	1	−3	3	2

In the table above, if $h(x) = g(f(x))$, then $h'(2) =$

(A) −18
(B) −6
(C) 2
(D) 4

$$h'(x) = g'(f(x)) \cdot f'(x)$$
$$h'(2) = g'(f(2)) \cdot f'(2)$$
$$= g'(1) \cdot (-3)$$
$$= 6 \cdot (-3)$$
$$= -18$$

END OF PART A, SECTION I
IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY.
DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS BC

SECTION I, Part B

Time—45 Minutes

Number of questions—15

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test:

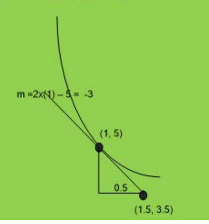
1. The **exact** numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

2. Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

$f(x+h) = f(x) + f'(x)h$ (Note: $h = 0.5$)

$x_0 = 1, x_{n+1} = x_n + 0.5$

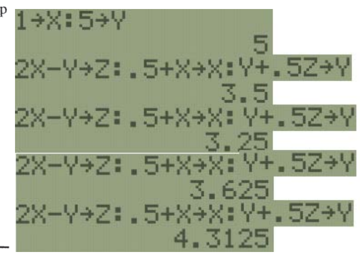
$y_0 = 5\ , y_{n+1} = y_n + 0.5y'_n$



x_0	y_0	y' = 2x - y
1	5	-3
$x_1 = x_0 + .5$	$y_1 = y_0 + .5(y')$	y' = 2x - y
1.5	3.5	-0.5
2	3.25	0.75
2.5	3.625	1.375
3	4.3125	1.6875

31. Let $y = f(x)$ be the solution to $\frac{dy}{dx} = 2x - y$ with $f(1) = 5$. Use Euler's Method to approximate $f(3)$, starting at $x = 1$ with a step size of 0.5.

(A) 3.625
(B) 3.75
(C) 3.844
(D) 4.313



32. The side of a square is changing at a constant rate of –0.4 meter per second. What is the rate of change of the perimeter of the square when the area of the square is 16 square meters?

(A) –3.2 meters per second
(B) –1.6 meters per second
(C) 1.6 meters per second
(D) 3.2 meters per second

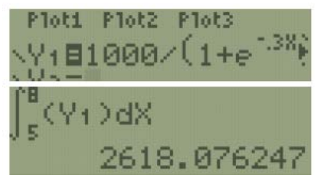
33. $\lim_{x \to 1} \frac{\sin 3x}{2x} =$

→ (A) 0.071
(B) 1.500
(C) 3
(D) The limit does not exist.

---

34. Profits, in dollars, for a small business can be modeled by $P(t) = \frac{1000}{1 + e^{\frac{-3t}{10}}}$, where t is measured in years. What are the total profits in years $5 \le t \le 8$ to the nearest dollar?

(A) \$2618
(B) \$2824
(C) \$5442
(D) \$5959



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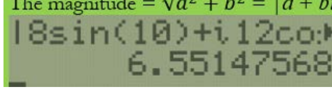
35. The position of a particle at any time $t \ge 0$ is given by $x(t) = 4\cos 2t$ and $y(t) = 3\sin 4t$. What is the particle's speed at time $t = 5$ seconds?

(A) 6.551
(B) 8.393
(C) 12.848
(D) 21.313

$a = x'(5) = -8\sin 2t\,|_{t=5} = -8\sin 10$
$b = y'(5) = 12\cos 4t\,|_{t=5} = 12\cos 20$
The magnitude $= \sqrt{a^2 + b^2} = |a + bi|$



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36. What is the minimum value of $f(x) = 2xe^{-x^2}$ on the closed interval $[-1, 2]$?

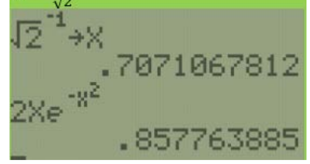
(A) -0.858
(B) -0.736
(C) 0.073
(D) 0.736

$f(x) = 2x\,e^{-x^2}$

$f'(x) = (2 - 4x^2)e^{-x^2} = 0$

$2x^2 - 1 = 0$

$x = \frac{1}{\sqrt{2}}$



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37. Which of the following integrals gives the area of the region enclosed by $r = 3 - 2\cos 4\theta$ on the closed interval $\left[0, \frac{\pi}{2}\right]$?

(A) $\int_0^{\frac{\pi}{2}} 3 - 2\cos 4\theta \, d\theta$

(B) $\int_0^{\frac{\pi}{2}} 8\sin 4\theta \, d\theta$

(C) $\frac{1}{2}\int_0^{\frac{\pi}{2}} (3 - 2\cos 4\theta)^2 \, d\theta$

(D) $\int_0^{\frac{\pi}{2}} (3 - 2\cos 4\theta)^2 \, d\theta$

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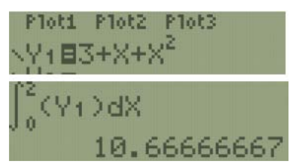
38. Use the Trapezoidal sum to approximate $\int_0^2 (3 + x + x^2)\,dx$ using $n = 4$ subintervals.

(A) 10.75
(B) 13.75
(C) 21.5
(D) 27.5



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39. A particle moves along the x-axis so that its acceleration at any time $t \geq 0$ is given by $a(t) = 8\sin 2t + 4\cos 2t$. If its initial velocity is 8 units per second and its initial position is $x = 4$, what is its position, in units, at time $t = 2$ seconds?

(A) 9.101
(B) 15.167
(C) 26.833
(D) 31.167

$v(t) = -4\cos 2t + 2\sin 2t + 12$
$x(t) = -2\sin 2t - \cos 2t + 12t + 5$

-2sin(4)-cos(4)▸
31.16724861

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40. $\int_0^1 x\left(\sqrt{x} + x^2\right) dx =$

(A) 0

(B) $\frac{13}{20}$

(C) $\frac{11}{12}$

(D) $\frac{11}{4}$

$\int_0^1 x^{\frac{3}{2}} + x^3\, dx$
$= \frac{2}{5}x^{\frac{5}{2}} + \frac{1}{4}x^4 \Big|_0^1 = \frac{2}{5} + \frac{1}{4} = \frac{13}{20}$

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41. $g(x) = \frac{d}{dx}\int_{-5}^{x^2} \cos t \, dt =$

(A) $\cos(x^2) \cdot 2x$

(B) $\cos(x^2)$

(C) $\sin(x^2) \cdot 2x$

(D) $\sin(x^2)$

$$\frac{d}{dx}\int_a^{h(x)} f(t)dt = f(x)h'(x)$$

So,

$$\frac{d}{dx}\int_{-5}^{x^2} \cos t \, dt = \cos x^2 \cdot (2x)$$

---

42. The region in the first quadrant bounded by $y = 4x - x^2$ and the x-axis is revolved around the x-axis. What is the volume of the resulting solid?

(A) 33.510
(B) 34.133
(C) 53.617
(D) 107.233

$y = 4x - x^2$

$\pi \int_0^4 y^2 dx =$

$\pi\int_0^4 ((4X-X^2)^2)dX$

107.2330292

---

43. What is the length of the curve $f(x) = \ln(x + 1)$ from $x = 1$ to $x = 3$?

(A) 1.614
(B) 1.870
(C) 2.121
(D) 2.320

$\int_1^3 \sqrt{1 + f'(x)^2}\, dx =$

$\int_1^3 (\sqrt{1+(1+X)^{-2}})dX$

2.120783012

GO ON TO THE NEXT PAGE.

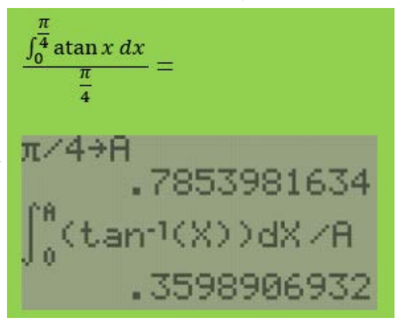
44. Let $f(x) = 3x - x^3$. What is the value of c on the closed interval [1, 3] that satisfies the conclusion of the Mean Value Theorem?

(A) 1.577
(B) 2
(C) 2.082
(D) 2.613

---

45. If $f(x) = \arctan x$, what is the average value of $f(x)$ on the closed interval $\left[0, \frac{\pi}{4}\right]$?

(A) 0.167
(B) 0.360
(C) 16.195
(D) 20.620



STOP

END OF PART B, SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART B ONLY.

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.

SECTION II, PART A

Time—30 minutes

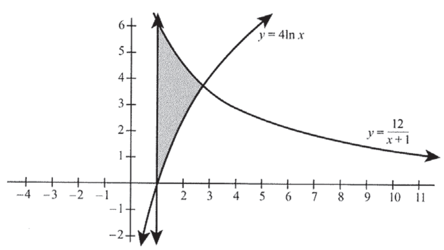
Number of problems—2

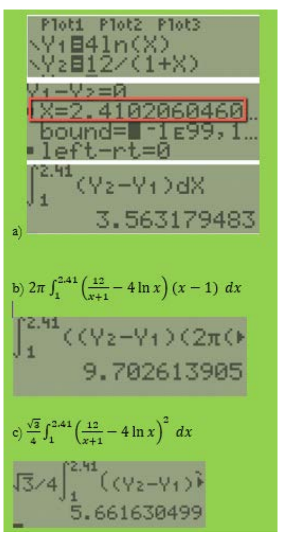
A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS

During the timed portion for Part A, you may work only on the problems in Part A.

On Part A, you are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

1. Let R be the region bounded by the graphs of $y = 4\ln x$, $y = \dfrac{12}{x+1}$, and $x = 1$, as shown in the figure below.





(a) Find the area of R.

(b) Find the volume of the solid that results when R is revolved about the vertical line $x = 1$.

(c) Let R be the base of a solid. Every cross-section of the solid perpendicular to the x-axis is an equilateral triangle with one side across the base. Find the volume of the solid.

2. For $t \ge 0$, a particle is moving along a curve with its position at time t given by $(x(t), y(t))$ and $\frac{dx}{dt} = 3t^2 - 4$, $\frac{dy}{dt} = 2t$. At time $t = 1$, its position is $(-3, 1)$.

(a) Find the slope of the particle's path at time $t = 3$.

(b) Find the x-coordinate of the particle's position at time $t = 5$.

(c) Find the speed of the particle at time $t = 5$.

(d) Find the acceleration vector of the particle at time $t = 5$.

a) $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2t}{3t^2-4}\Big|_{t=3} = \frac{6}{23}$

b) $-3 + \int_0^5 3t^2 - 4\ dt = 105$

```
-3+∫₁⁵(Y₁)dX
                105
```

c) speed $= \left|\frac{dx}{dt}, \frac{dy}{dt}\right|\Big|_{t=5} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\Big|_{t=5}$

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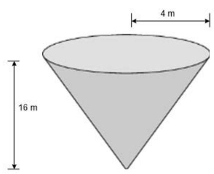
|Y₁(5)+iY₂(5)|
       71.70076708
```

d) $(6t, 2)|_{t=5} = (30, 2)$



SECTION II, PART B

Time—1 hour

Number of problems—4

NO CALCULATOR IS ALLOWED FOR THESE PROBLEMS

During the timed portion for Part B, you may continue to work on the problems in Part A without the use of any calculator.

3. A water storage tank is in the shape of an inverted cone whose height is 16 meters and radius is 4 meters. The tank initially contains 36π cubic meters. The water is filling the tank at a rate of $2\pi h^3$ cubic meters per minute.

(a) Find $\frac{dh}{dt}$ in terms of h.

(b) If $h = 12$ meters at time $t = 0$, find an equation for h as a function of t.

(c) At what time is the storage tank full?

a) $h = 4r$

$V = \frac{1}{3}\pi r^2 h = \frac{1}{48}\pi h^3$

$\frac{dV}{dt} = \frac{1}{16}\pi h^2 \frac{dh}{dt}$

$\frac{dV}{dt} = 2\pi h^3$

$2\pi h^3 = \frac{1}{16}\pi h^2 \frac{dh}{dt}$

$\frac{dh}{dt} = 32h$

b) $\frac{dh}{h} = 32dt$

$\ln h = 32t + \ln 12$

$h = 12e^{32t}$

c) $12e^{32t} = 16$

$e^{32t} = \frac{4}{3}$

$t = \frac{1}{32}\ln\frac{4}{3}$ (min)

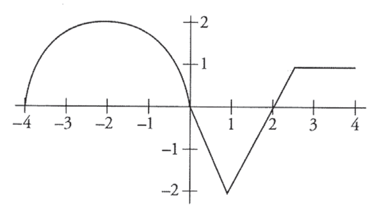
GO ON TO THE NEXT PAGE.

4. The function f is differentiable everywhere and the Maclaurin series is $\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{2n+1}=1-\frac{x^2}{3}+\frac{x^4}{5}-\frac{x^6}{7}+\ldots.$

(a) Use the Ratio Test to find the interval of convergence of the Maclaurin series of f.

(b) Approximate $f\left(\frac{1}{3}\right)$ using the first three nonzero terms.

(c) Write the first four nonzero terms and the general term for the Maclaurin series for $f'(x)$.



5. The figure above shows the graph of f, which is a continuous function defined on $-4 \le x \le 4$. The graph of f from $x = -4$ to $x = 0$ is a semicircle, and from $x = 0$ to $x = 4$ consists of three line segments. Let $g(x) = \int_0^x f(t)\,dt$.

(a) Find $g(2)$ and $g(-2)$.

(b) Evaluate $g'(1)$.

(c) Find all values of x where $g(x)$ has a point of inflection.

(d) At what value of x is $g(x)$ a relative minimum?

6. Bacteria is growing in a Petri dish at a rate that satisfies the differential equation $\frac{dA}{dt} = \frac{3}{10}(A - 40)$, where A is the number of bacteria and t is measured in days. There are initially 1200 bacteria in the dish.

(a) Use the line tangent to the graph of A at $t = 0$ to estimate the number of bacteria at the end of $t = 5$ days.

(b) Find $\frac{d^2A}{dt^2}$ in terms of A.

(c) Find the particular solution to the above differential equation with the initial condition $A(0) = 1200$.

a) $\frac{dA}{dt} = \frac{3}{10}(1200 - 40) = 348$

$1200 + 348(5) = 2940$

b) $\frac{d^2A}{dt^2} = \frac{3}{10}\frac{dA}{dt} = \frac{9}{100}(A - 40)$

c) $A - 40 = ke^{0.3t}$

$A = 1160e^{0.3t} + 40$

STOP

END OF EXAM

T3

T3

1. B	2. A	3. A	4. B	5. C	6. D	7. A	8. C	9. A	10. D
11. B	12. B	13. A	14. A	15. B	16. B	17. B	18. C	19. B	20. C
21. C	22. A	23. B	24. B	25. B	26. A	27. C	28. A	29. B	30. A
31. D	32. B	33. A	34. A	35. A	36. A	37. C	38. A	39. D	40. B
41. A	42. D	43. C	44. C	45. B					

T3