Section 1: MC
40 questions | 2 hrs 63%
Part A: 28 questions | 1 hr 20 mins | Calculator not permitted (approximately 44% of score).
Part B: 12 questions | 40 mins | Graphing calculator required for some questions (radian mode) (approximately 19% of score).

Section II: Free Response
4 Questions | 1hr | Approximately 37% of Score

Section I

Part A: Graphing calculator not permitted

1. What is the numerical coefficient of the term containing $x^3 y^2$ in the expansion of $(x + 2y)^5$?

   (A) 10

   (B) 20

   (C) 40

   (D) 80

2. What is the inverse of the matrix $\begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$?

   (A) $\begin{bmatrix} 3 & -1 \\ -2 & 4 \end{bmatrix}$

   (B) $\begin{bmatrix} -4 & 2 \\ 1 & -3 \end{bmatrix}$

   (C) $\begin{bmatrix} -\frac{4}{10} & \frac{2}{10} \\ \frac{1}{10} & -\frac{3}{10} \end{bmatrix}$

   (D) $\begin{bmatrix} \frac{3}{10} & -\frac{1}{10} \\ -\frac{2}{10} & \frac{4}{10} \end{bmatrix}$

3. If $f(x) = x^3 + A x^2 + Bx - 3$ and if $f(1) = 4$ and $f(-1) = -6$, what is the value of $2A + B$?

   (A) -2

   (B) 0

   (C) 8

   (D) 12

4. If $f(x) = x^3 - 2x - 1$, then $f(-2) =$

   (A) -17

   (B) -13

   (C) -5

   (D) -1

5. What is the domain of $f(x) = \frac{x - 5}{x^2 + 25}$?

   (A) All reals, $x \neq 5$

   (B) All reals, $x \neq \pm 5$

   (C) All reals, $x \neq -5$

   (D) All reals

6. What is the domain of $g(x) = \frac{\sqrt{x - 3}}{x^2 - 2x}$?

   (A) All reals, $x \neq 0, 2$

   (B) All reals, $x \geq 3, x \neq 0, 2$

   (C) All reals, $x \geq 3$

   (D) All reals

7. Which of the following represents the equation of $f^{-1}(x)$ for the inverse of the function $f(x) = 4 e^{3x-9}$?

   (A) $f^{-1}(x) = \frac{\ln\left(\frac{x}{4}\right) + 9}{3}$

   (B) $f^{-1}(x) = \frac{1}{4 e^{3x-9}}$

   (C) $f^{-1}(x) = \frac{1}{4} e^{-3x+9}$

   (D) $f^{-1}(x) = \frac{\ln(4x) - 9}{3}$

8. Which of the following is equivalent to the expression $\log \sqrt[3]{\dfrac{a^2 b}{c}}$?

   (A) $\dfrac{^3\, 2\log a + \log b}{\log c}$

   (B) $\dfrac{1}{3}\left(\dfrac{2\log a + \log b}{\log c}\right)$

   (C) $\dfrac{1}{3}(2\log a + \log b - \log c)$

   (D) $\sqrt[3]{2}\log a + \log b - \log c$

9. Identify the vertical asymptote(s) for the function $f(x) = \dfrac{x^2 - 4}{x^3 + 2x^2}$.

   (A) $x = -2,\ x = 0$

   (B) $x = -2,\ x = 2$

   (C) $x = 0,\ x = 2$

   (D) $x = 0$

10. Given $f(x) = \begin{cases} -\dfrac{1}{3}x^2 + 9, & x \le 0 \\ (x-3)^2, & x > 0 \end{cases}$, find $f(-3)$.

    (A) 36

    (B) 10

    (C) 8

    (D) 6

11. Find the slant asymptote of $f(x) = \dfrac{x^2 + 2x - 1}{x - 1}$

    (A) $y = 1$

    (B) $y = x - 1$

    (C) $y = x + 1$

    (D) $y = x + 3$

12. In polar coordinates, which of the following choices is *not* equivalent to $\left(2,\ \dfrac{5\pi}{6}\right)$?

    (A) $\left(-2,\ -\dfrac{\pi}{6}\right)$

    (B) $\left(-2,\ \dfrac{11\pi}{6}\right)$

    (C) $\left(2,\ -\dfrac{11\pi}{6}\right)$

    (D) $\left(2,\ -\dfrac{7\pi}{6}\right)$

13. Which of the following represents zeros of $r = 2 - 4\sin 2\theta$?

    (A) $\dfrac{\pi}{6},\ \dfrac{5\pi}{6}$

    (B) $0,\ \dfrac{\pi}{2},\ \pi,\ \dfrac{3\pi}{2}$

    (C) $\dfrac{\pi}{4},\ \dfrac{3\pi}{4},\ \dfrac{5\pi}{4},\ \dfrac{7\pi}{4}$

    (D) $\dfrac{\pi}{12},\ \dfrac{5\pi}{12},\ \dfrac{13\pi}{12},\ \dfrac{17\pi}{12}$

14. Which of the following choices represents the corresponding rectangular equation of the curve with the parametric equations $x(t) = t^3$, $y(t) = 1 - t$?

    (A) $y^3 - 3y^2 + 3y + x - 1 = 0$

    (B) $y^3 - 3y^2 - 3y - x - 1 = 0$

    (C) $y^3 + x - 1 = 0$

    (D) $y^3 - x - 1 = 0$

15. Evaluate: $\cos\left[\arctan\left(-\dfrac{2}{3}\right)\right]$, given that $\dfrac{\pi}{2} \le \theta \le \pi$

    (A) $-\dfrac{3\sqrt{13}}{13}$

    (B) $-\dfrac{2\sqrt{13}}{13}$

    (C) $\dfrac{2\sqrt{13}}{13}$

    (D) $\dfrac{3\sqrt{13}}{13}$

16. Given that $\sin\theta = -\dfrac{1}{5}$ and $\tan\theta > 0$, find $\cos\theta$.

    (A) $\dfrac{\sqrt{26}}{5}$

    (B) $\dfrac{2\sqrt{6}}{5}$

    (C) $-\dfrac{2\sqrt{6}}{5}$

    (D) $-\dfrac{\sqrt{26}}{5}$

17. Which of the following is a possible equation for the sinusoidal graph shown below?
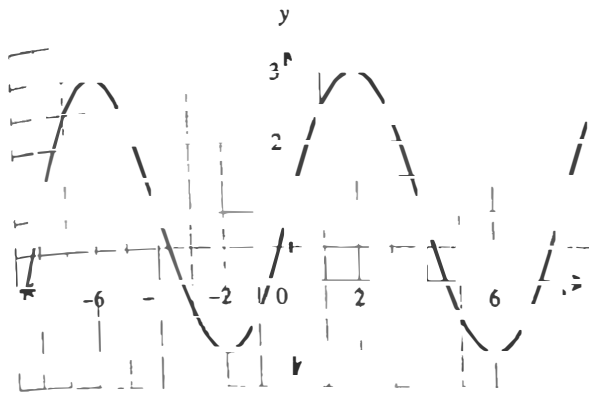
(A) $y = 2\sin(x) + 1$

(B) $y = 2\cos\left(\frac{\pi}{4}x\right) + 1$

(C) $y = 2\sin\left(\frac{\pi}{4}x + 1\right)$

(D) $y = 2\sin\left(\frac{\pi}{4}x\right) + 1$

18. Which of the following is equivalent to $\sin 5x \cos 3x - \cos 5x \sin 3x$?

(A) $\cos 8x$

(B) $\sin 8x$

(C) $\cos 2x$

(D) $\sin 2x$

19. Given $\sin x = -\frac{1}{8}$ and $\tan x < 0$, find $\sin 2x$.

(A) $-\frac{\sqrt{65}}{32}$

(B) $-\frac{1}{4}$

(C) $-\frac{3\sqrt{7}}{32}$

(D) $\frac{3\sqrt{7}}{32}$

20. Determine the period of the function
$$y = \frac{1}{2}\sin\left(\frac{1}{3}x - \pi\right)$$

(A) $\frac{1}{2}$

(B) $\frac{2\pi}{3}$

(C) 3π

(D) 6π

21. Solve the equation $\log_a 3 - \log_a b = c$ for b.

(A) $\frac{c}{3a}$

(B) $\frac{3}{a^c}$

(C) $\frac{a}{3c}$

(D) a^{3c}

22. Which of the following statements is true about the function $g(x) = x^7 + x^3 + \sin x$?

(A) The function is even and symmetric with respect to the origin.

(B) The function is even and symmetric with respect to the y-axis.

(C) The function is odd and symmetric with respect to the origin.

(D) The function is odd and symmetric with respect to the y-axis.

23. Which is an equation for the linear function f that satisfies the conditions $f(-3) = -7$ and $f(5) = -11$?

(A) $y + 7 = -\frac{1}{2}(x + 3)$

(B) $y + 11 = \frac{1}{2}(x - 5)$

(C) $y - 7 = -\frac{1}{2}(x - 3)$

(D) $y - 11 = -\frac{1}{2}(x + 5)$

24. Given the functions $f(x) = x^2 - 4$ and $g(x) = \sqrt{x} + 4$, determine the composition $(g \cdot f)(x)$.

(A) $x^4 - 8x^2 + 12$

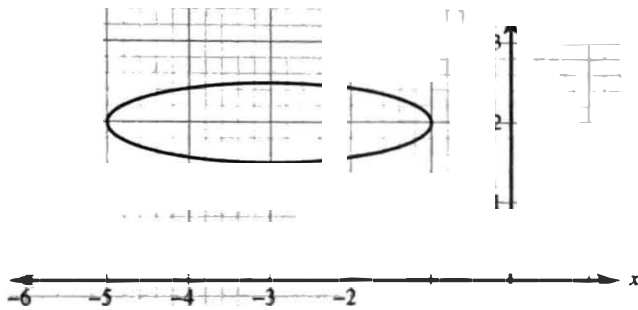(B) $\sqrt{x^2 - 4} + 4$

(C) $x + 8\sqrt{x} + 12$

(D) $x + 12$

25. Given the function $g(x) = -(12x - 7)^2(34x + 89)^3$, determine $\lim_{x \to -\infty} g(x)$.

(A) $-\infty$
(B) 0
(C) 1
(D) ∞

26. Solve the inequality: $\dfrac{2(x-1)}{(x+1)(x-3)} \le 0$

(A) $(-\infty, -1] \cup [1, 3]$
(B) $[-1, 1] \cup [3, \infty)$
(C) $(-\infty, -1) \cup [1, 3)$
(D) $(-1, 1) \cup (3, \infty)$

27. Which of the following choices represents the equation of the ellipse shown in the graph below?

(A) $(x+3)^2 + 16(y-2)^2 = 4$
(B) $16(x+3)^2 + (y-2)^2 = 4$
(C) $(x+3)^2 - 16(y-2)^2 = 4$
(D) $16(x+3)^2 - (y-2)^2 = 4$
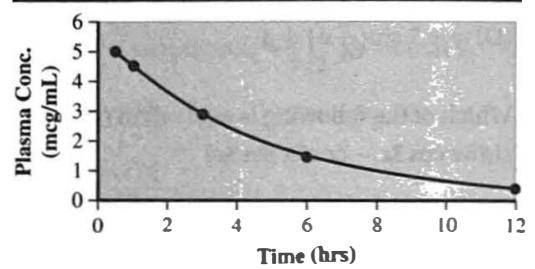
28. Evaluate $\lim_{x \to \infty} -5^{-x} - 2$.

(A) $-\infty$
(B) -2
(C) 0
(D) ∞

29. A patient is given an injection of a new medication known as Drug A. Below is the information provided on the effects of Drug A on the patient.
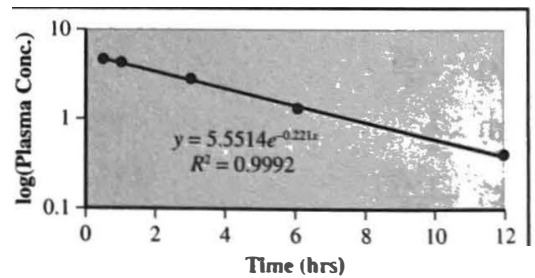
Time (hrs)	Plasma Concentration (μg/mL)
0.5	5.0
1	4.5
3	2.9
6	1.4
12	0.4

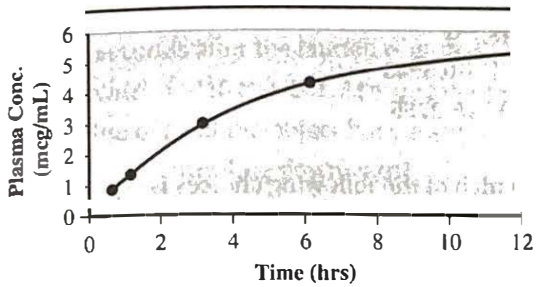Which of the following statements are equivalent?

I. The graph of the time elapsed and plasma concentration on the Cartesian plane

II. The graph of the time elapsed and plasma concentration on a semilog plot

$y = 5.5514e^{-0.221x}$
$R^2 = 0.9992$

III. The graph of the time elapsed and plasma concentration on a semilog plot

Plasma Conc. (mcg/mL) vs. Time (hrs)

(A) I, II, and III

(B) I and III only

(C) I and II only

(D) II and III only

30. What is the function whose graph is a reflection over the y-axis of the graph of $f(x) = 1 - 3^x$?

(A) $g(x) = 1 - 3^{-x}$

(B) $g(x) = 1 + 3^x$

(C) $g(x) = 3^x - 1$

(D) $g(x) = \log_3(x - 1)$

31. Which of the following functions does *not* have an inverse function?

(A) $y = \sin x \left(-\frac{\pi}{2} \le x \le \frac{\pi}{2} \right)$

(B) $y = x^3 + 2$

(C) $y = \frac{x}{x + 2}$

(D) $y = \frac{1}{2} e^x$

32. Give an algebraic expression for $\cos(\sin^{-1}(x))$.

(A) $\sqrt{1 - x^2}$

(B) $\sqrt{1 + x^2}$

(C) $\sqrt{x^2 - 1}$

(D) $1 - x^2$

33. A circle is graphed using the parametric equations shown below.

$$x = 5\cos(t) + 3$$

$$y = 5\sin(t) - 1$$

Where is the center of the circle located?

(A) $(-3, -1)$

(B) $(-3, 1)$

(C) $(3, -1)$

(D) $(3, 1)$

34. The table shows the predicted growth of a particular bacteria after various numbers of hours. Write an explicit formula for the sequence of the number of bacteria.

| Hours (n)	1	2	3	4	5
Number of Bacteria (a_n)	19	38	57	76	95

(A) $a_n = 19n + 19$

(B) $a_n = n + 19$

(C) $a_n = \frac{1}{19}n$

(D) $a_n = 19n$

35. What are the points where the graph of the polynomial $f(x) = 7(x - 5)(x + 5)^2$ passes through the x-axis?

(A) $x = 5$ only

(B) $x = -5$ only

(C) $x = -5$ and $x = 5$

(D) Nowhere

36. Evaluate: $\lim\limits_{x \to \infty} \dfrac{3x^2 - 9x - 4}{5x^2 - 4x + 8}$

(A) Does not exist

(B) 0

(C) $\frac{3}{5}$

(D) $\frac{9}{4}$

Part B: Graphing calculator required

37. Given the graph of the function $y = f(x)$ shown below with points $a, b, c, d, e, f, g,$ and h, which of the following pairs of points below has the greatest average rate of change?
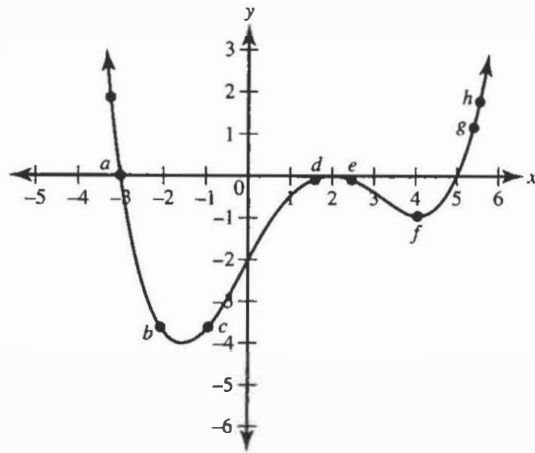
(A) a and c

(B) b and c

(C) c and d

(D) f and h

38. Given the function $f(x) = (x^2 - 5)(x - 3)(3x - 2)$, at what value of x is the absolute maximum of $f(x)$ over the interval $[-2.28, 3.25]$?

(A) -2.28

(B) -1.27

(C) 1.35

(D) 3.25

39. Approximate $\log_a 24$, given that $\log_a 2 \approx 0.4307$ and $\log_a 3 \approx 0.6826$.

(A) 1.9747

(B) 1.1133

(C) 0.8820

(D) 0.2940

40. Solve for x: $3^{2x} = 5^{x-1}$

(A) -2.7381

(B) -1

(C) -0.5563

(D) 15.2755

41. Which of the following choices is equivalent to the complex number $3 - 8i$?

(A) $\sqrt{73}\left(\cos\frac{37\pi}{60} + i\sin\frac{37\pi}{60}\right)$

(B) $\sqrt{73}\left(\cos\frac{97\pi}{60} + i\sin\frac{97\pi}{60}\right)$

(C) $\sqrt{55}\left(\cos\frac{37\pi}{60} + i\sin\frac{37\pi}{60}\right)$

(D) $\sqrt{55}\left(\cos\frac{97\pi}{60} + i\sin\frac{97\pi}{60}\right)$

42. Find all solutions in the interval $[0, 2\pi]$ for the trigonometric equation $6\cos^2 x - 5\sin x - 2 = 0$.

(A) $-1.333333, -4.4749, \frac{\pi}{6}, \frac{5\pi}{6}$

(B) $\frac{\pi}{6}, \frac{5\pi}{6}$

(C) $2, 5.1416$

(D) $\frac{7\pi}{6}, \frac{11\pi}{6}$

43. Find the angle between two vectors $\vec{p}$ and $\vec{q}$ with magnitude 2 and 1, respectively, that has a dot product equal to $\sqrt{3}$.

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{4}$

(D) $\frac{\pi}{6}$

44. How long will it take for $1,000 to double in an investment when interest is compounded continuously at the rate of 5.8% per annum? Round your answer to the nearest year.

(A) 11 years

(B) 12 years

(C) 13 years

(D) 14 years

45. An object is launched straight up from ground level with an initial velocity of 50 feet per second. The height, h (in feet above ground level), of the object t seconds after the launch is given by the function $h(t) = -16t^2 + 50t$. At approximately what value of t will the object have a height of 28 feet and be traveling downward?

(A) 0.73 seconds
(B) 1.56 seconds
(C) 1.84 seconds
(D) 2.39 seconds

46. Suppose you drop a tennis ball from a height of 15 feet. After the ball hits the floor, it rebounds to 85% of its previous height. How high, to the nearest tenth, will the ball rebound after its third bounce?

(A) 1.9 feet
(B) 7.8 feet
(C) 9.2 feet
(D) 10.8 feet

47. Let $f(x) = -2x^6 + 15x^5$. For what values of x does the graph of f have a point of inflection?

I. $x = 0$
II. $x = 5$
III. $x = \dfrac{15}{2}$

(A) I only
(B) I and II only
(C) I, II, and III
(D) none of these

48. After performing analyses on a set of data, Joseph examined the scatterplot of the residual values for each analysis. Which scatterplot indicates the best linear fit for the data?

(A)

(B)

(C)

(D)
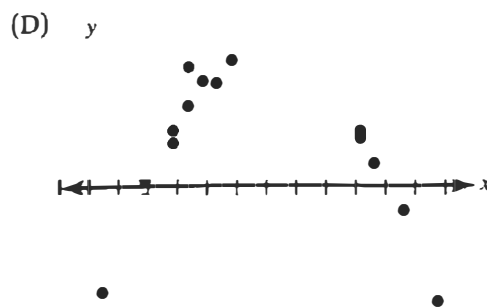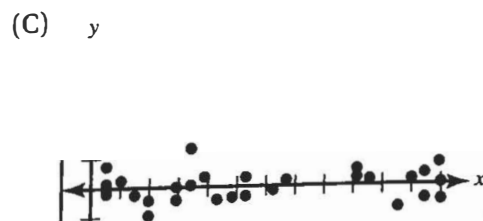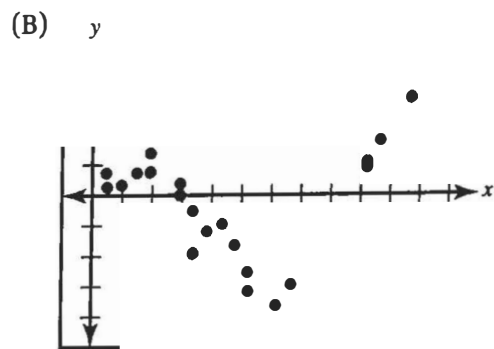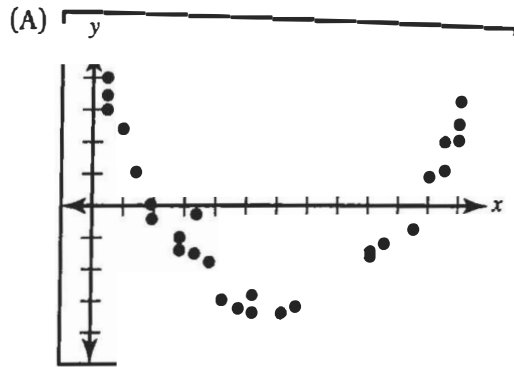
Section II

Part A: Graphing calculator required

1. A marketing company was hired to examine the data of the sales in a single year of a local coffee shop to see where improvements could be made to increase profits. The data of their most popular hot and cold brew beverages are given in the tables below.

Hot Brew Coffee Beverages

Yearly Quarter, x	1	2	3	4
Sales (dollars), h	36,208	37,620	39,088	40,594

Cold Brew Coffee Beverages

Yearly Quarter, x	1	2	3	4
Sales (dollars), c	42,000	37,800	34,020	27,556

(a) Write a linear regression model, $h(x)$, for the hot brew coffee sales. Justify why a linear regression model is an appropriate fit for the data.

(b) Write an exponential regression model, $c(x)$, for the cold brew coffee sales, rounding all coefficients to the nearest thousandth. Justify why an exponential regression model is an appropriate fit for the data.

(c) What would be a reason for the linear regression model to be increasing while the exponential regression model is decreasing during the year?

(d) Using the model found in part (a), predict the sales, to the nearest dollar, halfway between quarters 3 and 4. Comment on the reasonableness of the prediction.

(e) Using the model found in part (b), predict the sales, to the nearest dollar, for the second quarter of the following year. Comment on the reasonableness of the prediction.

2. A company is trying to design packaging for its new product and is deciding between two different designing options.
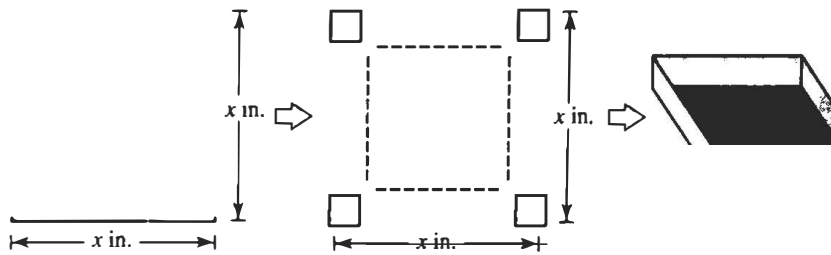
Option 1:

An open box with the dimensions labeled in the figure below. All units are inches.
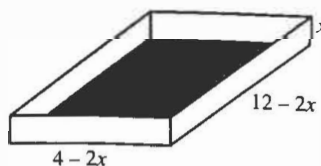
Option 2:

An open box that is made from a square sheet of cardboard by cutting out 4-inch squares from each corner as shown below and then folding along the dotted lines.

The company wants to ensure that both options can hold a product that has a volume of 36 cubic inches.

(a) If possible, find the dimensions of the box in Option 1. If it is not possible, explain why.

(b) If possible, find the dimensions of the box in Option 2. If it is not possible, explain why.

(c) The company decides that it is going to build a rectangular section to hold all of the products in the warehouse. If the company has 400 feet of fencing, what is the area of the largest rectangular section it can enclose if it uses the building as one side of the section?

(d) The company also decides to examine the costs involved with shipping the product and derives the following piecewise function that determines the cost of shipping, $c(p)$, per pound, p:

$$c(p) = \begin{cases} ap + 3, p < 100 \\ p^2 + 2p, 100 \le p \le 2,000 \end{cases}$$

Find the cost of shipping if the weight is 100 pounds.

(e) Using the above piecewise function, for what values of a would the cost of shipping be a continuous function when $p = 100$?

(f) The shipping truck moves along the highway in such a way that its velocity vector is given by $\vec{v}(t) = \langle e^{2t}, \ln(t + 1) \rangle$, $0 \le t \le 3$, where distance is measured in mile and time is measured in hours. At $t = 2$ hours, describe the direction the particle is moving and find the speed of the particle to the nearest hundredth.

Part B: Graphing calculator is not permitted

3. Given the function $f(x) = -(7x - 14)^2(10x + 80)^3$, do the following.

    (a) State the degree of f.
    (b) Find the zeros of f along with their multiplicities.
    (c) State the values of x for which the graph of f is tangent to the x-axis, and justify your answer.
    (d) Evaluate $\lim\limits_{x \to -\infty} f(x)$ and $\lim\limits_{x \to \infty} f(x)$.
    (e) Using parts (a)–(d), include a sketch of the graph of $f(x)$. The axes need not be scaled.

4. The function f is defined by $f(x) = \dfrac{3x + 6}{2 - x}$, where $x \neq 2$.

    (a) Find the equation of the vertical asymptote of the graph of f.
    (b) Find the equation of the horizontal asymptote of the graph of f.
    (c) Find the coordinates where the graph of f crosses the x-axis.
    (d) Find the coordinates where the graph of f crosses the y-axis.
    (e) Sketch the graph of f on the axes below.
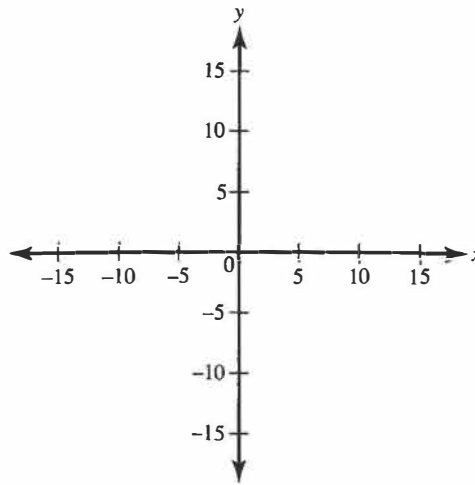
    (f) The function g is defined by $g(x) = \dfrac{ax + 6}{2 - x}$, where $x \neq 2$. Given that $g(x) = g^{-1}(x)$, find the value of a.

Answer Key

1.	C		25.	D
2.	D		26.	C
3.	C		27.	A
4.	C		28.	B
5.	D		29.	C
6.	B		30.	A
7.	A		31.	C
8.	C		32.	A
9.	D		33.	C
10.	D		34.	D
11.	D		35.	A
12.	C		36.	C
13.	D		37.	D
14.	A		38.	D
15.	A		39.	A
16.	C		40.	A
17.	D		41.	B
18.	D		42.	B
19.	C		43.	D
20.	D		44.	B
21.	B		45.	D
22.	C		46.	C
23.	A		47.	B
24.	B		48.	C