Section II

Part A: Graphing calculator required

1. A marketing company was hired to examine the data of the sales in a single year of a local coffee shop to see where improvements could be made to increase profits. The data of their most popular hot and cold brew beverages are given in the tables below.

Hot Brew Coffee Beverages

Yearly Quarter, x	1	2	3	4
Sales (dollars), h	36,208	37,620	39,088	40,594

Cold Brew Coffee Beverages

Yearly Quarter, x	1	2	3	4
Sales (dollars), c	42,000	37,800	34,020	27,556

(a) Write a linear regression model, $h(x)$, for the hot brew coffee sales. Justify why a linear regression model is an appropriate fit for the data.

(b) Write an exponential regression model, $c(x)$, for the cold brew coffee sales, rounding all coefficients to the nearest thousandth. Justify why an exponential regression model is a appropriate fit for the data.

(c) What would be a reason for the linear regression model to be increasing while the exponential regression model is decreasing during the year?

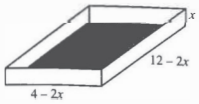
(d) Using the model found in part (a), predict the sales, to the nearest dollar, halfway between quarters 3 and 4. Comment on the reasonableness of the prediction.

(e) Using the model found in part (b), predict the sales, to the nearest dollar, for the second quarter of the following year. Comment on the reasonableness of the prediction.

2. A company is trying to design packaging for its new product and is deciding between two different designing options.

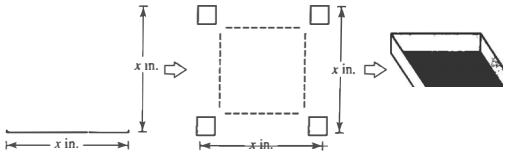
Option 1:

An open box with the dimensions labeled in the figure below. All units are inches.



Option 2:

An open box that is made from a square sheet of cardboard by cutting out 4-in squares from each corner as shown below and then folding along the dotted lines.



The company wants to ensure that both options can hold a product that has a volume of 36 cubic inches.

(a) If possible, find the dimensions of the box in Option 1. If it is not possible, explain why.
(b) If possible, find the dimensions of the box in Option 2. If it is not possible, explain why.
(c) The company decides that it is going to build a rectangular section to hold all of the products in the warehouse. If the company has 400 feet of fencing, what is the area of the largest rectangular section it can enclose if it uses the building as one side of the section?
(d) The company also decides to examine the costs involved with shipping the product and derives the following piecewise function that determines the cost of shipping, $c(p)$, per pound, p:

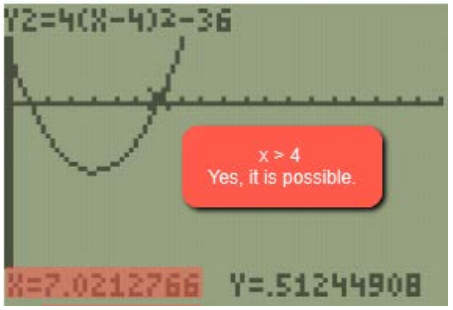
$$c(p) = \begin{cases} ap + 3, p < 100 \\ p^2 + 2p, 100 \le p \le 2,000 \end{cases}$$

Find the cost of shipping if the weight is 100 pounds.

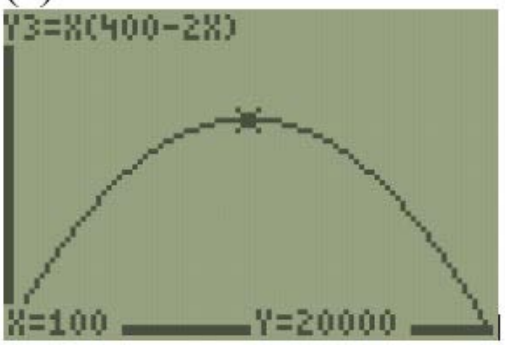
(e) Using the above piecewise function, for what values of a would the cost of shipping be a continuous function when $p = 100$?
(f) The shipping truck moves along the highway in such a way that its velocity vector is given by $\vec{v}(t) = \langle e^{2t}, \ln(t+1) \rangle$, $0 \le t \le 3$, where distance is measured in mile and time is measured in hours. At $t = 2$ hours, describe the direction the particle is moving and find the speed of the particle to the nearest hundredth.

(a)

(b)



(c)



(d) $100^2 + 2 \times 100 = 10{,}200$

(e) $100a + 3 = 10{,}200$

$a = 101.97$

(e)

$v(2) = < e^4, \ln(3) >$

To find the length: