AP Precalculus Exam

The sample exam questions that follow illustrate the relationship between the course framework and the AP Precalculus Exam and serve as examples of the types of questions that appear on the exams. After the sample questions is a table which shows which skill, learning objective(s), and unit each question relates to. The table also provides the answers to the multiple-choice questions.

Section I: Multiple-Choice

PART A

No calculator is allowed for this part of the exam.

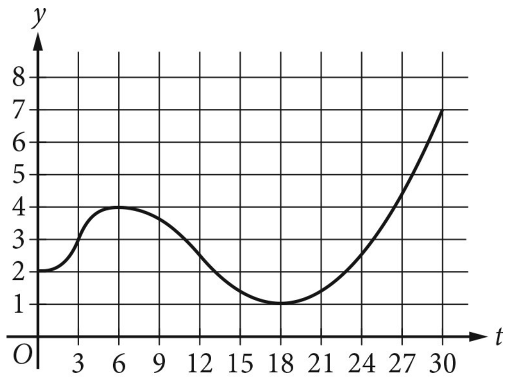
1. The polynomial function p is given by $p(x) = -4x^5 + 3x^2 + 1$. Which of the following statements about the end behavior of p is true?

    (A) The sign of the leading term of p is positive, and the degree of the leading term of p is even; therefore, $\lim_{x \to -\infty} p(x) = \infty$ and $\lim_{x \to \infty} p(x) = \infty$.

    (B) The sign of the leading term of p is negative, and the degree of the leading term of p is odd; therefore, $\lim_{x \to -\infty} p(x) = \infty$ and $\lim_{x \to \infty} p(x) = -\infty$.

    (C) The sign of the leading term of p is positive, and the degree of the leading term of p is odd; therefore, $\lim_{x \to -\infty} p(x) = -\infty$ and $\lim_{x \to \infty} p(x) = \infty$.

    (D) The sign of the leading term of p is negative, and the degree of the leading term of p is odd; therefore, $\lim_{x \to -\infty} p(x) = -\infty$ and $\lim_{x \to \infty} p(x) = \infty$.



Graph of W

2. The depth of water, in feet, at a certain place in a lake is modeled by a function W. The graph of $y = W(t)$ is shown for $0 \le t \le 30$, where t is the number of days since the first day of a month. What are all intervals of t on which the depth of water is increasing at a decreasing rate?

(A) $(3, 6)$ only

(B) $(3, 12)$

(C) $(0, 3)$ and $(18, 30)$ only

(D) $(0, 6)$ and $(18, 30)$

3. Which of the following functions has a zero at $x = 3$ and has a graph in the xy-plane with a vertical asymptote at $x = 2$ and a hole at $x = 1$?

(A) $h(x) = \dfrac{x^2 - 4x + 3}{x^2 - 3x + 2}$

(B) $j(x) = \dfrac{x^2 - 5x + 6}{x^2 - 3x + 2}$

(C) $k(x) = \dfrac{x - 3}{x^2 - 3x + 2}$

(D) $m(x) = \dfrac{x - 3}{x^2 - 4x + 3}$

4. The polynomial function p is an odd function. If $p(3) = -4$ is a relative maximum of p, which of the following statements about $p(-3)$ must be true?

(A) $p(-3) = 4$ is a relative maximum.

(B) $p(-3) = -4$ is a relative maximum.

(C) $p(-3) = 4$ is a relative minimum.

(D) $p(-3) = -4$ is a relative minimum.

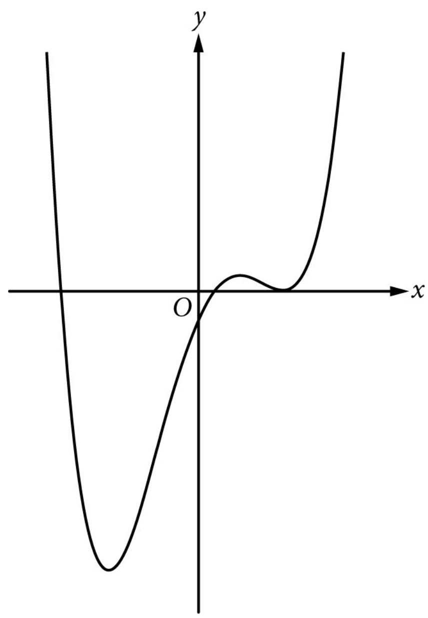
5. The function g is given by $g(x) = x^3 - 3x^2 - 18x$, and the function h is given by $h(x) = x^2 - 2x - 35$. Let k be the function given by $k(x) = \dfrac{h(x)}{g(x)}$. What is the domain of k ?

(A) all real numbers x where $x \neq 0$

(B) all real numbers x where $x \neq -5,\ x \neq 7$

(C) all real numbers x where $x \neq -3,\ x \neq 0,\ x \neq 6$

(D) all real numbers x where $x \neq -5,\ x \neq -3,\ x \neq 0,\ x \neq 6,\ x \neq 7$



Graph of g

6. The figure shown is the graph of a polynomial function g. Which of the following could be an expression for $g(x)$?

(A) $0.25(x-5)(x-1)(x+8)$

(B) $0.25(x+5)(x+1)(x-8)$

(C) $0.25(x-5)^2(x-1)(x+8)$

(D) $0.25(x+5)^2(x+1)(x-8)$

x	-8	-4	-2	-1	0	3
$f(x)$	87	55	5	-4	-7	20

7. The table gives values for a polynomial function f at selected values of x. Let $g(x) = af(bx) + c$, where a, b, and c are positive constants. In the xy-plane, the graph of g is constructed by applying three transformations to the graph of f in this order: a horizontal dilation by a factor of 2, a vertical dilation by a factor of 3, and a vertical translation by 5 units. What is the value of $g(-4)$?

(A) 266

(B) 170

(C) 28

(D) 20

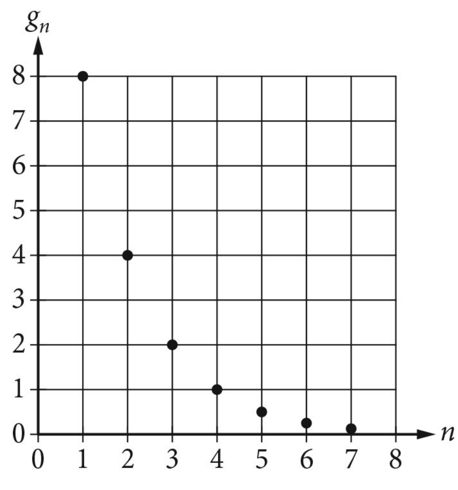
8. Let k, w, and z be positive constants. Which of the following is equivalent to $\log_{10}\left(\frac{kz}{w^2}\right)$?

(A) $\log_{10}(k+z) - \log_{10}(2w)$

(B) $\log_{10} k + \log_{10} z - 2\log_{10} w$

(C) $\log_{10} k + \log_{10} z - \frac{1}{2}\log_{10} w$

(D) $\log_{10} k - \log_{10} z + 2\log_{10} w$



9. Values of the terms of a geometric sequence g_n are graphed in the figure. Which of the following is an expression for the nth term of the geometric sequence?

(A) $g_n = 4\left(\frac{1}{2}\right)^{(n-2)}$

(B) $g_n = 8(2)^{(n-1)}$

(C) $g_n = 8\left(\frac{1}{2}\right)^{n}$

(D) $g_n = 16\left(\frac{1}{2}\right)^{(n-1)}$

x	$g(x)$
-2	4
0	$\frac{1}{2}$
3	-2
4	3
36	9

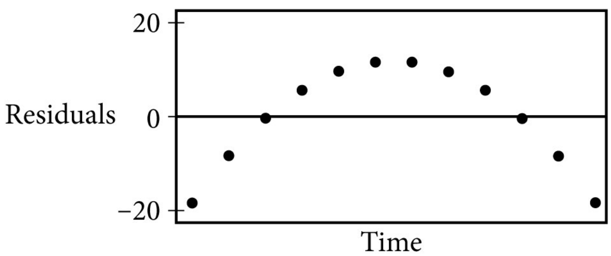
10. The table gives values of the function g for selected values of x. The function f is given by $f(x) = 3^x + x^2$. What is the value of $f(g(3))$?

(A) -72

(B) $\frac{37}{9}$

(C) 9

(D) 97



11. A food vendor developed a new sandwich type for sale. The vendor made estimates about the sales of the new sandwich type over time. A linear regression was used to develop a model for the sales over time. The figure shows a graph of the residuals of the linear regression. Which of the following statements about the linear regression is true?

(A) The linear model is not appropriate, because there is a clear pattern in the graph of the residuals.

(B) The linear model is not appropriate, because the graph of the residuals has more points above 0 than below 0.

(C) The linear model is appropriate, because there is a clear pattern in the graph of the residuals.

(D) The linear model is appropriate, because the positive residual farthest from 0 and the negative residual farthest from 0 are about the same distance, although more points are above 0 than below 0.

12. The value, in millions of dollars, of transactions processed by an online payment platform is modeled by the function M. The value is expected to increase by 6.1% each quarter of a year. At time $t = 0$ years, 54 million dollars of transactions were processed. If t is measured in years, which of the following is an expression for $M(t)$? (Note: A quarter is one fourth of a year.)

(A) $54(0.061)^{(t/4)}$

(B) $54(0.061)^{(4t)}$

(C) $54(1.061)^{(t/4)}$

(D) $54(1.061)^{(4t)}$

13. Iodine-131 has a half-life of 8 days. In a particular sample, the amount of iodine-131 remaining after d days can be modeled by the function h given by $h(d) = A_0(0.5)^{(d/8)}$, where A_0 is the amount of iodine-131 in the sample at time $d = 0$. Which of the following functions k models the amount of iodine-131 remaining after t hours, where A_0 is the amount of iodine-131 in the sample at time $t = 0$? (There are 24 hours in a day, so $t = 24d$.)

(A) $k(t) = A_0(0.5)^{(t/24)}$

(B) $k(t) = A_0\left(0.5^{(1/24)}\right)^{(8t)}$

(C) $k(t) = A_0\left(0.5^{(24)}\right)^{(t/8)}$

(D) $k(t) = A_0\left(0.5^{(1/192)}\right)^{t}$

14. What are all values of x for which $\ln\left(x^3\right) - \ln x = 4$?

(A) $x = -2$ and $x = 2$

(B) $x = -e^2$ and $x = e^2$

(C) $x = e^2$ only

(D) $x = e^4$

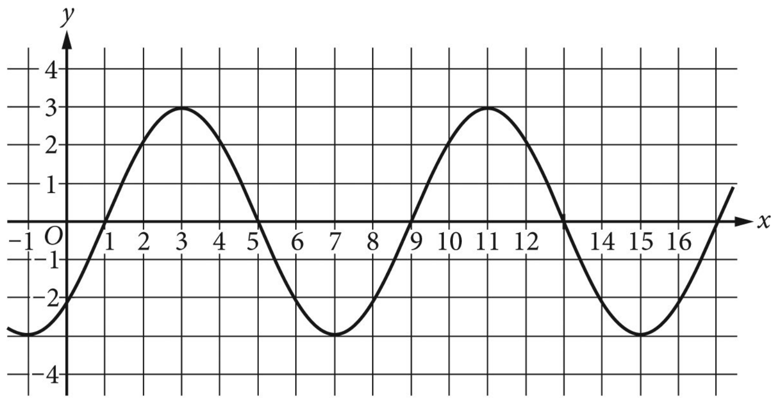
15. Let $f(x) = 1 + 3\sec x$ and $g(x) = -5$. In the xy-plane, what are the x-coordinates of the points of intersection of the graphs of f and g for $0 \le x < 2\pi$?

(A) $x = \frac{\pi}{3}$ and $x = \frac{5\pi}{3}$

(B) $x = \frac{\pi}{6}$ and $x = \frac{5\pi}{6}$

(C) $x = \frac{2\pi}{3}$ and $x = \frac{4\pi}{3}$

(D) $x = \frac{7\pi}{6}$ and $x = \frac{11\pi}{6}$



Graph of g

16. The figure shows the graph of a sinusoidal function g. What are the values of the period and amplitude of g ?

(A) The period is 4, and the amplitude is 3.

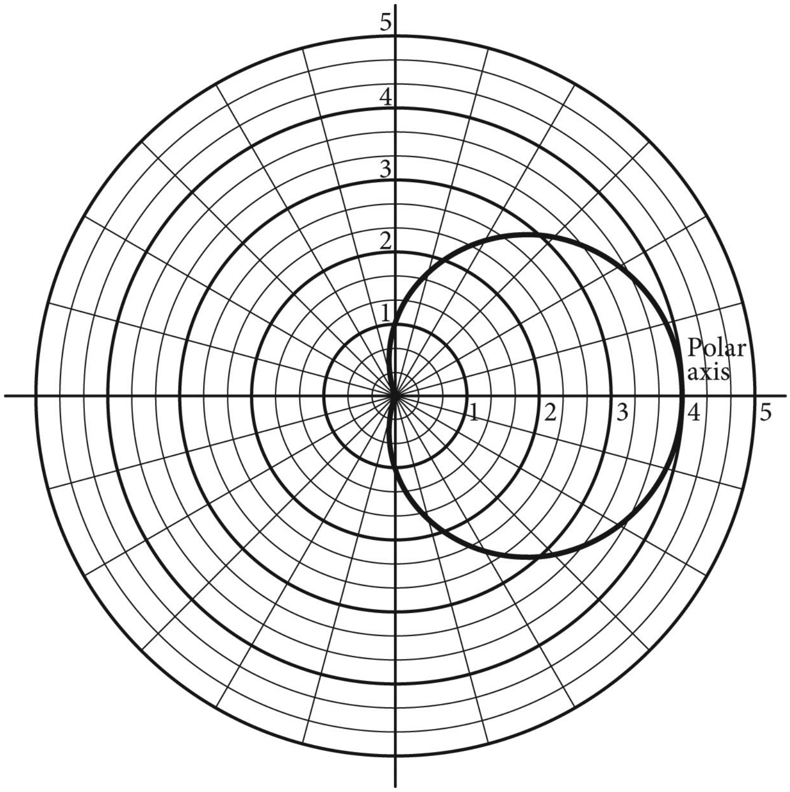
(B) The period is 8, and the amplitude is 3.

(C) The period is 4, and the amplitude is 6.

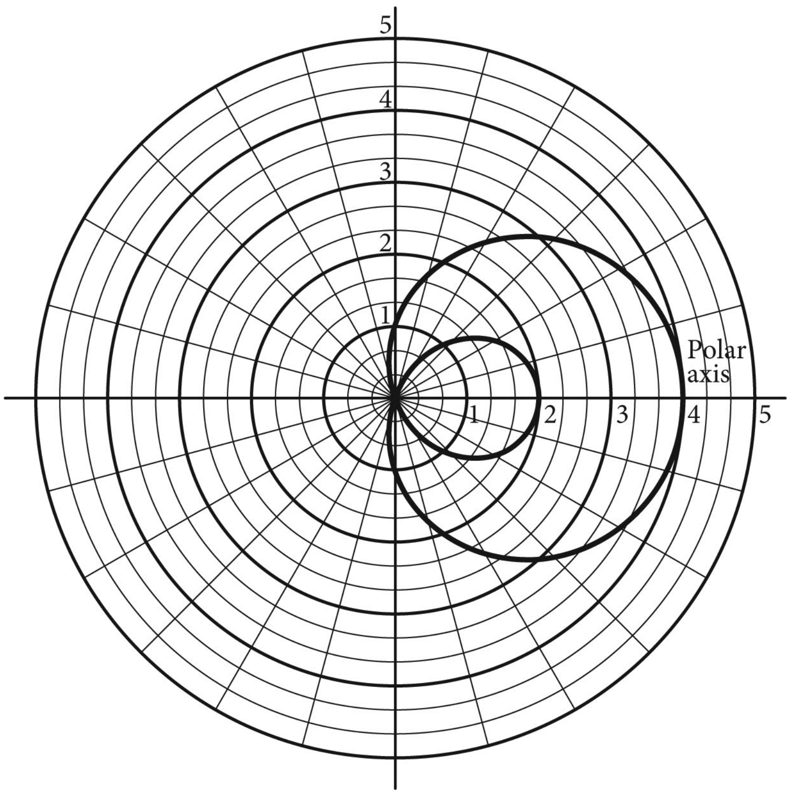
(D) The period is 8, and the amplitude is 6.

17. Which of the following is the graph of the polar function $r = f(\theta)$, where $f(\theta) = 3\cos\theta + 2$, in the polar coordinate system for $0 \le \theta \le 2\pi$?

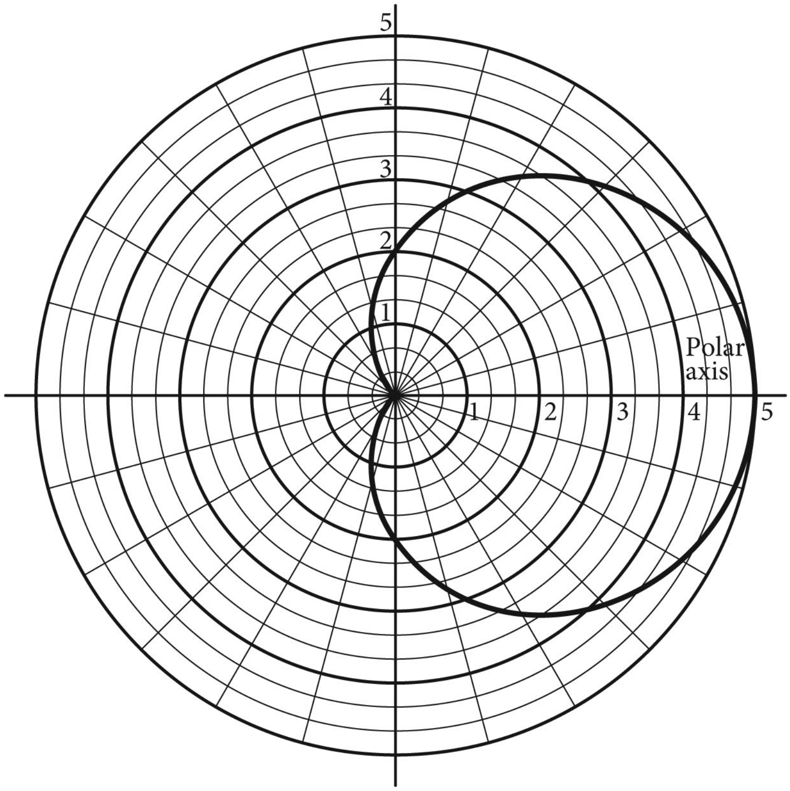
(A)



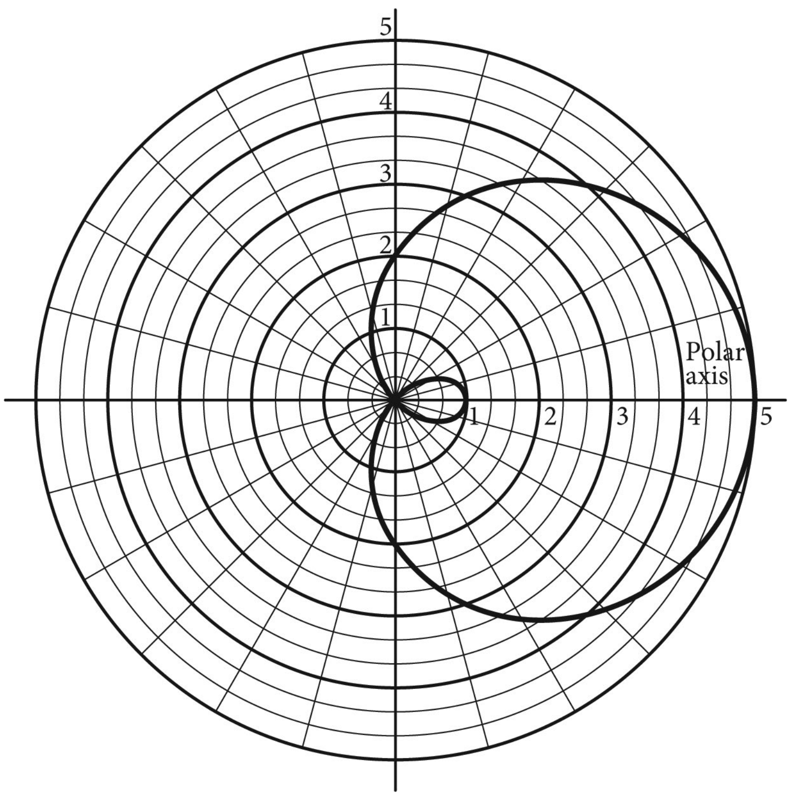
(B)



(C)



(D)



18. What are all values of $\theta, -\pi \le \theta \le \pi$, for which $2\cos\theta > -1$ and $2\sin\theta > \sqrt{3}$?

(A) $-\frac{5\pi}{6} < \theta < \frac{5\pi}{6}$

(B) $\frac{\pi}{6} < \theta < \frac{5\pi}{6}$ only

(C) $-\frac{2\pi}{3} < \theta < \frac{2\pi}{3}$ only

(D) $\frac{\pi}{3} < \theta < \frac{2\pi}{3}$ only

19. A polar function is given by $r = f(\theta) = -1 + \sin\theta$. As θ increases on the interval $0 < \theta < \frac{\pi}{2}$, which of the following is true about the points on the graph of $r = f(\theta)$ in the xy-plane?

(A) The points on the graph are above the x-axis and are getting closer to the origin.

(B) The points on the graph are above the x-axis and are getting farther from the origin.

(C) The points on the graph are below the x-axis and are getting closer to the origin.

(D) The points on the graph are below the x-axis and are getting farther from the origin.

PART B

A graphing calculator is required for some questions on this part of the exam.

20. The temperature, in degrees Celsius (°C), in a city on a particular day is modeled by the function T defined by $T(t) = \dfrac{75t^3 - 836t^2 + 3100t - 4185}{14t^2 + 10t - 35}$, where t is measured in hours from 12 P.M. for $2 \le t \le 9$. Based on the model, how many hours did it take for the temperature to increase from 0°C to 5°C?

(A) 7.701

(B) 5.420

(C) 4.114

(D) 2.280

x	$f(x)$
−2	10
−1	15
1	40
2	56

21. The table presents values for a function f at selected values of x. An exponential regression $y = ab^x$ is used to model these data. What is the value of $f(1.5)$ predicted by the exponential function model?

(A) 46.767

(B) 47.342

(C) 47.800

(D) 47.917

22. The number of minutes of daylight per day for a certain city can be modeled by the function D given by $D(t) = 160\cos\left(\frac{2\pi}{365}(t - 172)\right) + 729$, where t is the day of the year for $1 \le t \le 365$. Which of the following best describes the behavior of $D(t)$ on day 150?

(A) The number of minutes of daylight per day is increasing at a decreasing rate.

(B) The number of minutes of daylight per day is decreasing at a decreasing rate.

(C) The number of minutes of daylight per day is increasing at an increasing rate.

(D) The number of minutes of daylight per day is decreasing at an increasing rate.

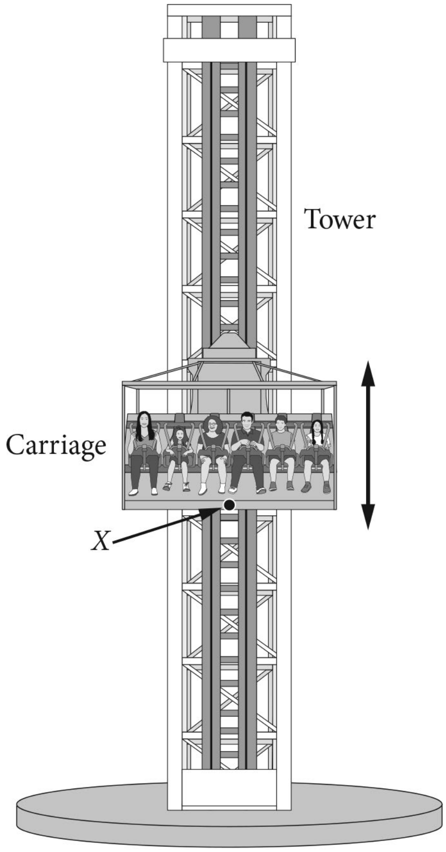
23. The function g is given by $g(x) = \sin x - \cos x$ and has a period of 2π. In order to define the inverse function of g, which of the following specifies a restricted domain for g and provides a rationale for why g is invertible on that domain?

(A) $0 \le x \le \pi$, because all possible values of $g(x)$ occur without repeating on this interval.

(B) $-\frac{\pi}{4} \le x \le \frac{3\pi}{4}$, because all possible values of $g(x)$ occur without repeating on this interval.

(C) $0 \le x \le \pi$, because the length of this interval is half of the period.

(D) $-\frac{\pi}{4} \le x \le \frac{3\pi}{4}$, because the length of this interval is half of the period.



Note: Figure not drawn to scale.

24. A theme park thrill ride involves a tower and a carriage that rapidly moves passengers up and down along a vertical axis, as shown in the figure. The carriage is lifted to the top of the tower, then released to move down the tower. The ride involves 10 controlled bounces from the highest point to the lowest point, and back to the highest point. A point X is located on the bottom of the carriage. The height of X above the ground, in feet, can be modeled by a periodic function H. At time $t = 0$ seconds, X is at its highest point of 120 feet. The lowest point for X is at a height of 20 feet. The next time X is at its highest point is at time $t = 8$ seconds, which is the end of the first bounce. Which of the following can be an expression for $H(t)$, where t is the time in seconds?

(A) $50\sin\left(\frac{\pi}{4}t\right) + 70$

(B) $50\cos\left(\frac{\pi}{4}t\right) + 70$

(C) $50\sin\left(\frac{\pi}{8}t\right) + 70$

(D) $50\cos\left(\frac{\pi}{8}t\right) + 70$

Section II: Free-Response

The following are examples of the kinds of free-response questions found on the exam.

PART A

A graphing calculator is required for these questions.

x	1	2	3	4	5
$f(x)$	-10	-5	4	17	34

1. Let f be an increasing function defined for $x \geq 0$. The table gives values of $f(x)$ at selected values of x. The function g is given by $g(x) = \dfrac{x^3 - 14x - 27}{x + 2}$.

(A) (i) The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(5)$ as a decimal approximation, or indicate that it is not defined.

(ii) Find the value of $f^{-1}(4)$, or indicate that it is not defined.

(B) (i) Find all values of x, as decimal approximations, for which $g(x) = 3$, or indicate there are no such values.

(ii) Determine the end behavior of g as x decreases without bound. Express your answer using the mathematical notation of a limit.

(C) (i) Use the table of values of $f(x)$ to determine if f is best modeled by a linear, quadratic, exponential, or logarithmic function.

(ii) Give a reason for your answer based on the relationship between the change in the output values of f and the change in the input values of f.

2. Students who completed a class participated in a year-long study to see how much content from the class they retained over the following year. At the end of the class, students completed an initial test to determine the group's content knowledge. At that time ($t = 0$), the group of students achieved a score of 75 out of 100 points. For the next 12 months, the group was evaluated at the end of each month to track their retention of the content. After 3 months ($t = 3$), the group's score was 70.84 points.

The group's score can be modeled by the function R given by $R(t) = a + b\ln(t + 1)$, where $R(t)$ is the score, in points, for month t, and t is the number of months since the initial test.

(A) (i) Use the given data to write two equations that can be used to find the values for constants a and b in the expression for $R(t)$.

(ii) Find the values for a and b.

(B) (i) Use the given data to find the average rate of change of the scores, in points per month, from $t = 0$ to $t = 3$ months. Express your answer as a decimal approximation. Show the computations that lead to your answer.

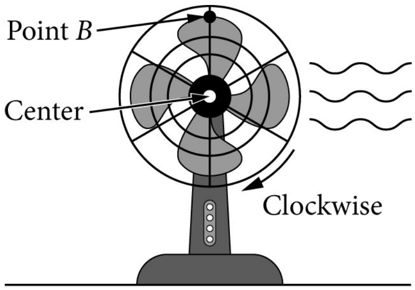
(ii) Interpret the meaning of your answer from (i) in the context of the problem.

(iii) Consider the average rates of change of R from $t = 3$ to $t = p$ months, where $p > 3$. Are these average rates of change less than or greater than the average rate of change from $t = 0$ to $t = 3$ months found in (i)? Explain your reasoning.

(C) The leaders of the study decide to use model R to make predictions about the group's score beyond 12 months (1 year). For a given year, model R is an appropriate model if the group's predicted score at the end of the year is at least 1 point lower than the group's predicted score at the end of the previous year. Based on this information, for how many years is model R an appropriate model? Give a reason for your answer. (Note: The end of a year occurs every 12 months from the initial evaluation—$t = 12, t = 24, \ldots$)

PART B

No calculator is allowed for these questions.

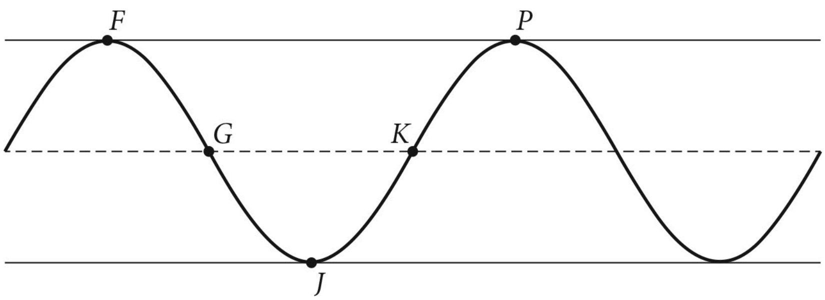


Note: Figure not drawn to scale.

3. The blades of an electric fan rotate in a clockwise direction and complete 5 rotations every second. Point B is on the tip of one of the fan blades and is located directly above the center of the fan at time $t = 0$ seconds, as indicated in the figure. Point B is 6 inches from the center of the fan. The center of the fan is 20 inches above a level table on which the fan sits. As the fan blades rotate at a constant speed, the distance between B and the surface of the table periodically decreases and increases.

   The sinusoidal function h models the distance between B and the surface of the table, in inches, as a function of time t in seconds.

   (A) The graph of h and its dashed midline for two full cycles is shown. Five points, F, G, J, K, and P are labeled on the graph. No scale is indicated, and no axes are presented.
   Determine possible coordinates $(t, h(t))$ for the five points: F, G, J, K, and P.



   (B) The function h can be written in the form $h(t) = a\sin(b(t + c)) + d$. Find values of constants a, b, c, and d.

   (C) Refer to the graph of h in part (A). The t-coordinate of K is t_1, and the t-coordinate of P is t_2.

   (i) On the interval (t_1, t_2), which of the following is true about h ?

   a. h is positive and increasing.
   b. h is positive and decreasing.
   c. h is negative and increasing.
   d. h is negative and decreasing.

   (ii) Describe how the rate of change of h is changing on the interval (t_1, t_2).

4. Directions:

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

(A) The functions g and h are given by

$$g(x) = 3\ln x - \frac{1}{2}\ln x$$
$$h(x) = \frac{\sin^2 x - 1}{\cos x}.$$

(i) Rewrite $g(x)$ as a single natural logarithm without negative exponents in any part of the expression. Your result should be of the form $\ln(\text{expression})$.

(ii) Rewrite $h(x)$ as an expression in which $\cos x$ appears once and no other trigonometric functions are involved.

(B) The functions j and k are given by

$$j(x) = 2(\sin x)(\cos x) - \cos x$$
$$k(x) = 8e^{(3x)} - e.$$

(i) Solve $j(x) = 0$ for values of x in the interval $\left[0, \frac{\pi}{2}\right]$.

(ii) Solve $k(x) = 3e$ for values of x in the domain of k.

(C) The function m is given by

$$m(x) = \cos(2x) + 4.$$

Find all input values in the domain of m that yield an output value of $\frac{9}{2}$.

Answer Key

1. B
2. A
3. A
4. C
5. C
6. C
7. D
8. B
9. A
10. B
11. A
12. D
13. D
14. C
15. C
16. B
17. D
18. D
19. C
20. D
21. A
22. A
23. B
24. B