Trigonometric Functions and Identities

Pythagorean Theorem:		$a^2 + b^2 = c^2$	
Special Right Triangles:		30° - 60° - 90° 45° - 45° - 90°	$x, x\sqrt{3}, 2x$ $x, x, x\sqrt{2}$
Law of Sines:	$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$	Heron's Formula:	$A = \sqrt{s(s-a)(s-b)(s-c)}$
Law of Cosines:	$a^2 = b^2 + c^2 - 2bc \cos A$	$b^2 = a^2 + c^2 - 2ac \cos B$	$c^2 = a^2 + b^2 - 2ab \cos C$
Linear Velocity:	$v = r \bullet \omega$ $v = r \bullet \frac{\theta}{t}$	Angular Velocity:	$\omega = \frac{\theta}{t}$
Reciprocal Identities:	$\sin\theta = \frac{1}{\csc\theta}$ $\csc\theta = \frac{1}{\sin\theta}$	$\cos\theta = \frac{1}{\sec\theta}$ $\sec\theta = \frac{1}{\cos\theta}$	$\tan\theta = \frac{1}{\cot\theta}$ $\cot\theta = \frac{1}{\tan\theta}$
Pythagorean Identities:	$\sin^2\theta + \cos^2\theta = 1$	$1 + \tan^2\theta = \sec^2\theta$	$1 + \cot^2\theta = \csc^2\theta$
Sum & Difference Identities:	$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$	$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$	
Double-Angle Identities:	$\sin 2\theta = 2\sin\theta\cos\theta$ $\cos 2\theta = 2\cos^2\theta - 1$	$\cos 2\theta = \cos^2\theta - \sin^2\theta$ $\cos 2\theta = 1 - 2\sin^2\theta$	

Sequences and Series

The n^{th} Term of an Arithmetic Sequence:	$a_n = a_1 + (n-1)d$	The n^{th} Term of a Geometric Sequence:	$a_n = a_1 r^{n-1}$
Sum of a Finite Arithmetic Series:	$\sum_{k=1}^{n} a_k = \frac{n}{2}(a_1 + a_n)$	$S_n = \frac{n}{2}\left[2a_1 + (n-1)d\right]$	
Sum of a Finite Geometric Series:	$\sum_{k=1}^{n} a_k = \frac{a_1(1-r^n)}{1-r}, \ \lvert r\rvert \neq 1$	$S_n = \frac{a_1 - a_n r}{1-r}, \ \lvert r\rvert \neq 1$	
Sum of an Infinite Geometric Series:	$\sum_{n=1}^{\infty} a_n = \frac{a_1}{1-r}, \ \lvert r\rvert < 1$		
Binomial Theorem:	$(a+b)^n = {}_nC_0 a^n b^0 + {}_nC_1 a^{n-1} b^1 + {}_nC_2 a^{n-2} b^2 + \cdots + {}_nC_r a^{n-r} b^r + \cdots + {}_nC_n a^0 b^n$		
Permutations:	${}_nP_r = \frac{n!}{(n-r)!}$	Combinations:	${}_nC_r = \frac{n!}{(n-r)!r!}$

Projectile Motion

Vertical Position:	$y = tv_0 \sin\theta - \frac{1}{2}gt^2 + h_0$	Horizontal Distance:	$x = tv_0 \cos\theta$
Vertical Free-Fall Motion:	$s(t) = -\frac{1}{2}gt^2 + v_0 t + s_0$	$v(t) = -gt + v_0$	$g \approx 32\frac{ft}{\sec^2} \approx 9.8\frac{m}{\sec^2}$

Conic Sections

Circle:	Standard Form:	$(x-h)^2+(y-k)^2=r^2$	
Parabola:	Standard Form:	$(x-h)^2=4p(y-k)$	$(y-k)^2=4p(x-h)$
	Focus:	$(h, k+p)$	$(h+p, k)$
	Directrix:	$y=k-p$	$x=h-p$
Ellipse:	Standard Form:	$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$	$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$
	Foci:	$(h\pm c, k)$	$(h, k\pm c)$
	a, b, c Relationship:	$c^2=a^2-b^2$	$c^2=a^2-b^2$
Hyperbola:	Standard Form:	$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$	$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$
	Foci:	$(h\pm c, k)$	$(h, k\pm c)$
	Asymptotes:	$(y-k)=\pm\frac{b}{a}(x-h)$	$(y-k)=\pm\frac{a}{b}(x-h)$
	a, b, c Relationship:	$c^2=a^2+b^2$	$c^2=a^2+b^2$
	Eccentricity:	$e=\frac{c}{a}$	$e=\frac{c}{a}$

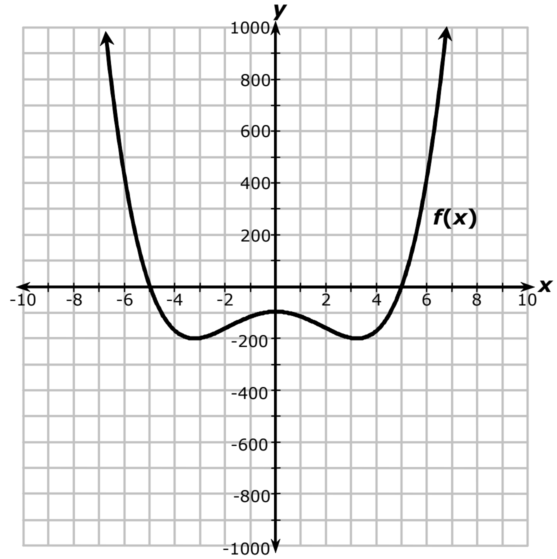
Exponential Functions

Simple Interest:	$I=prt$		
Compound Interest:	$A=P\left(1+\frac{r}{n}\right)^{nt}$	Continuous Compound Interest:	$A=Pe^{rt}$
Exponential Growth or Decay:	$N=N_0(1+r)^t$	Continuous Exponential Growth or Decay:	$N=N_0e^{kt}$

Coordinate Geometry

Distance Formula:	$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$		
Slope of a Line:	$m=\frac{y_2-y_1}{x_2-x_1}$		
Midpoint Formula:	$M=\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$		
Quadratic Equation:	$ax^2+bx+c=0$	Quadratic Formula:	$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
Slope-Intercept Form of a Line:	$y=mx+b$		
Point-Slope Form of a Line:	$y-y_1=m(x-x_1)$		
Standard Form of a Line:	$Ax+By=C$		

1 A polynomial function of degree four is graphed as shown.



Based on this graph, which statement is true?

A $f(x)$ has a total of four roots and three local extrema.

B $f(x)$ has a total of two roots and three local extrema.

C $f(x)$ has a total of two roots and five extrema.

D $f(x)$ has a total of four roots and five extrema.

2 The formula to calculate the level of sound intensity in decibels (dB) is

$$\beta = 10 \log\left(\frac{I}{I_0}\right)$$

where β is the number of decibels, I is the sound intensity in watts per square meter (W/m^2) of any sound, and $I_0 = 10^{-12}$ W/m^2, which is the intensity of the faintest sound audible to the human ear. The pain threshold for the human ear is 120 dB. What is the intensity, I, of this sound?

A 10^{120} W/m^2

B 10^{12} W/m^2

C 10^{0} W/m^2

D 10^{-12} W/m^2

3 Which function has a removable discontinuity at $x = 2$?

A $f(x) = \begin{cases} 3x - 1, x \leq 2 \\ x^2, x > 2 \end{cases}$

B $f(x) = \dfrac{x - 2}{x^2 - 9}$

C $f(x) = \dfrac{x^2 + 2x}{x^2 - 4}$

D $f(x) = \dfrac{x^2 + 2x - 8}{x^2 - 2x}$

4 Newton's Law of Cooling models the temperature, T, of an object over time, t, in minutes, using the function

$$T(t) = T_m + (T_0 - T_m)e^{-kt}$$

where T_0 is the initial temperature of the object, T_m is the temperature of the surrounding medium, and k is a constant for that particular medium. A cookie was taken from the oven and placed on a cooling rack with a surrounding air temperature of 20°C. After five minutes, the cookie cooled to 48°C. If $k = 0.35$, what was T_0, the initial temperature of the cookie?

A About 25°C

B About 60°C

C About 181°C

D About 276°C

5 What is the sum of the arithmetic series $\sum_{k=3}^{10} 5k - 12$?

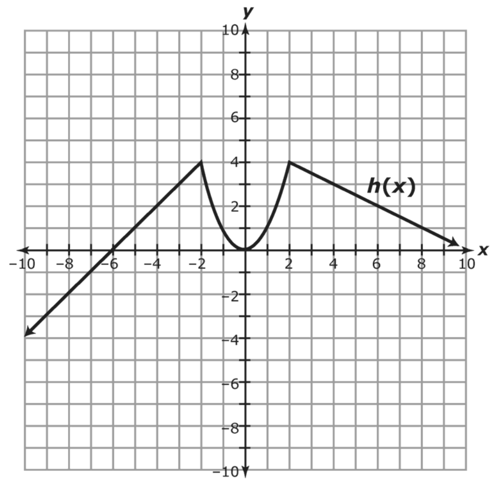
A 38

B 164

C 195

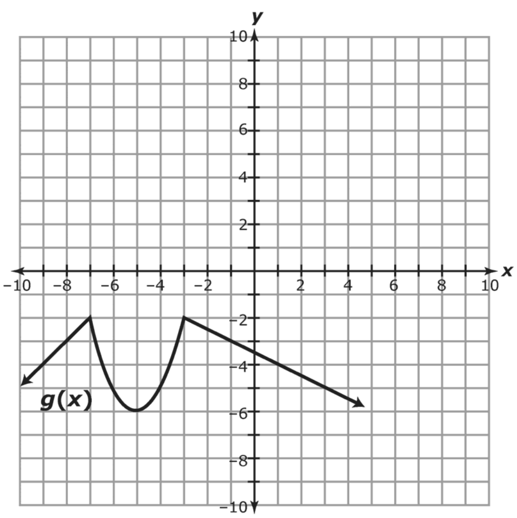
D 205

6 The graph of the function, $h(x)$, is shown.



Which graph represents the transformation $g(x) = h(x - 5) - 6$?

A



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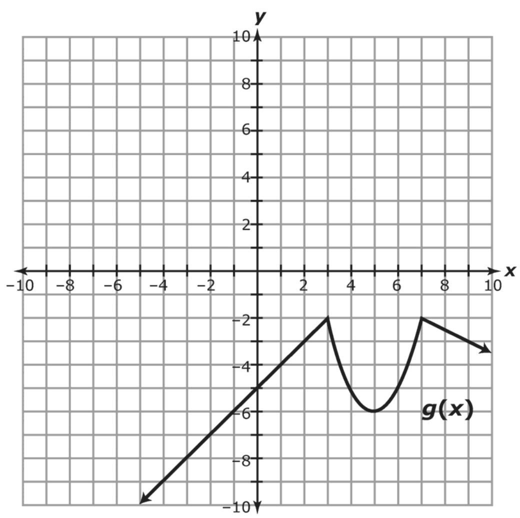

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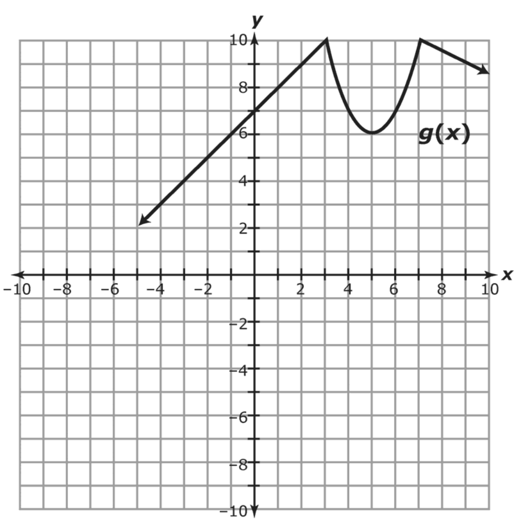

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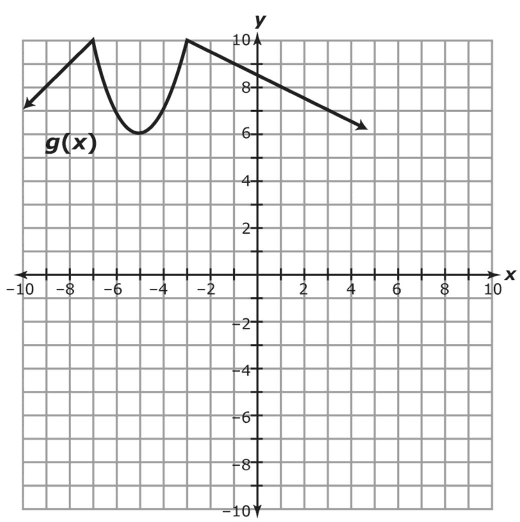


7 What is the sixth term in the expansion of $(x - 2y)^{10}$?

A $-13{,}440x^4y^6$

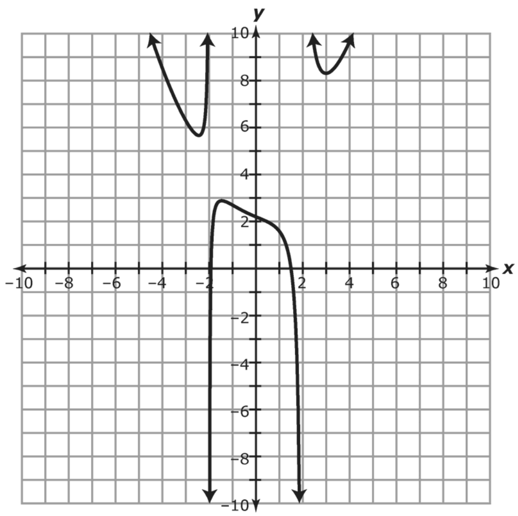
B $-8{,}064x^5y^5$

C $8{,}064x^5y^5$

D $13{,}440x^4y^6$

8 Which graph represents the function $f(x) = \dfrac{x^4 + 5x - 2}{3x^2 - 12} + 2$?

A



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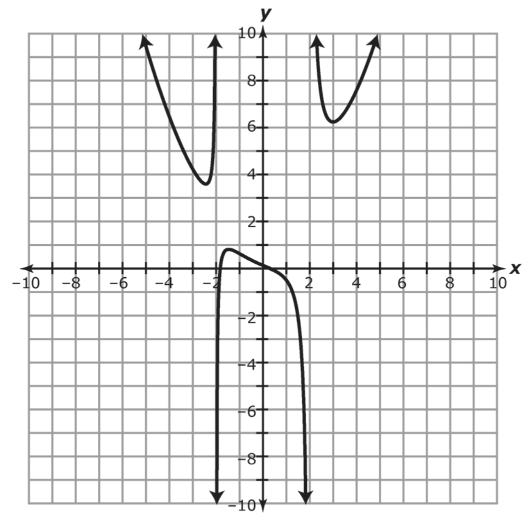

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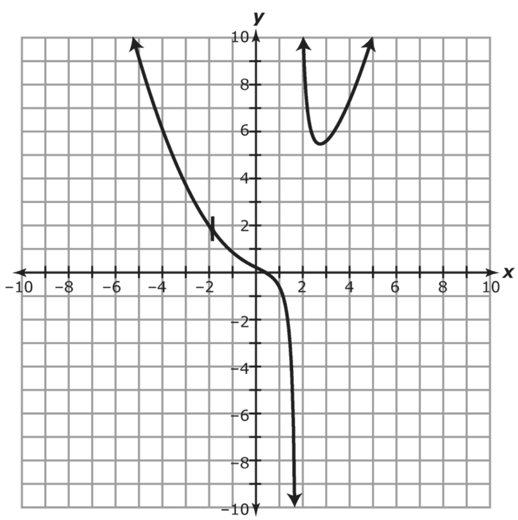

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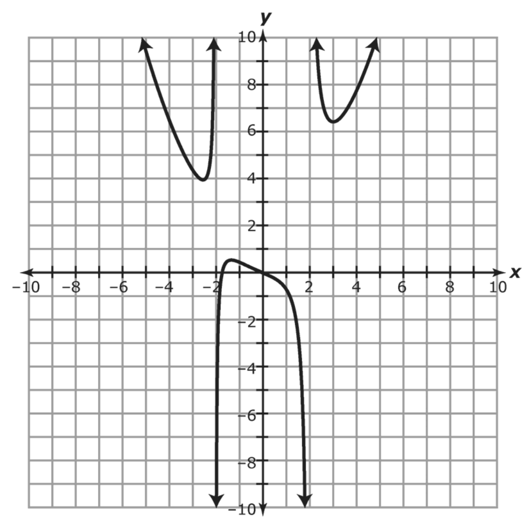
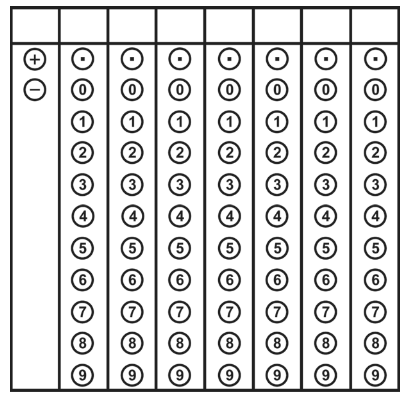


9 A ball is dropped from a height of 6 feet, and the return bounce is 82% of the previous height. How far has the ball traveled when it hits the ground for the sixth time? Round the answer to the nearest tenth of a foot.

Record the answer and fill in the bubbles on the grid provided. Be sure to use the correct place value.



10 If $f(x) = x^2 + 2x - 1$ and $g(x) = 3x$, what is $f(g(x))$?

A $3x^2 + 6x - 1$

B $6x^2 + 2x - 1$

C $9x^2 + 2x - 1$

D $9x^2 + 6x - 1$

11 Matthew and some friends are going to a concert. They hire a car service for $75 to drive them to a restaurant for dinner and then to the concert. They divide the $60 cost of the dinner equally. However, since Matthew’s dad provided concert tickets for the group, the friends agree that Matthew doesn’t have to help pay for the car service. The friends divide this cost equally among themselves. If each friend spends a total of $25, how many friends went to the concert with Matthew?

A 4

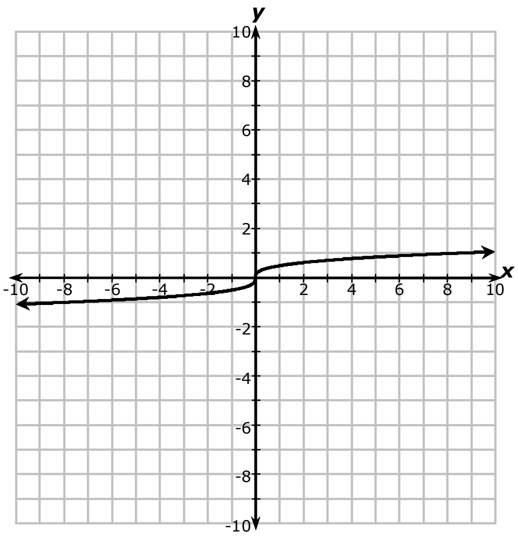
B 5

C 6

D 7

12 If $f(x) = 8x^3$, which graph represents $f^{-1}(x)$?

A



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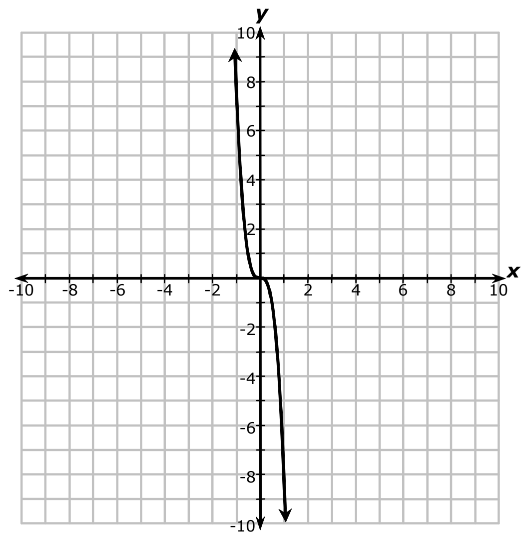

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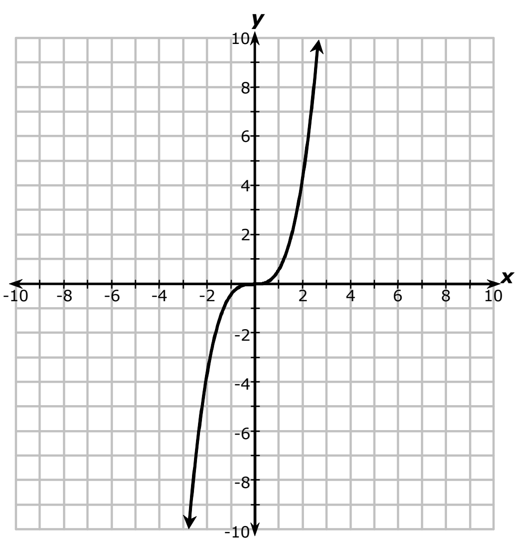

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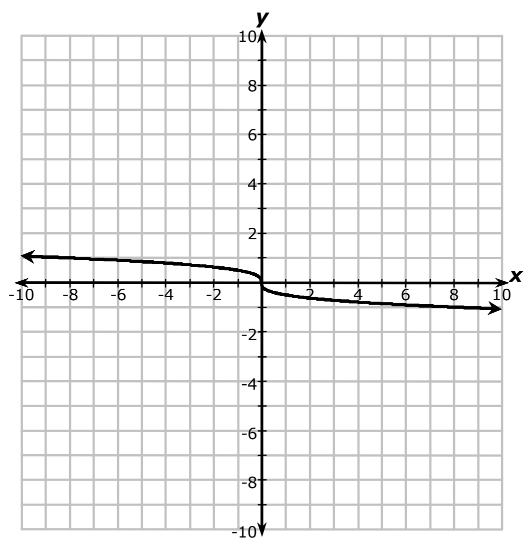


13 For an art project, Analy takes a 22 inch by 28 inch poster board and cuts congruent squares out of the corners. She folds the poster board along the cuts to create a container with no top. If the volume of Analy's container is 1,055 in^3, what is the approximate height of the container in inches?

A 1.02 inches

B 2.04 inches

C 2.99 inches

D 3.81 inches

14 Which equation represents a slant asymptote for the graph of the function $f(x) = \dfrac{2x^3 - 5x^2 - 8x + 1}{x^2 - x + 5}$?

A $y = 2x - 7$

B $y = 2x - 3$

C $y = x - 4$

D $y = x - 6$

15 Which list represents all possible rational zeros of the function $f(x) = 3x^3 - 8x^2 + 11x - 7$?

A $\pm 3, \pm 8, \pm 11, \pm 7$

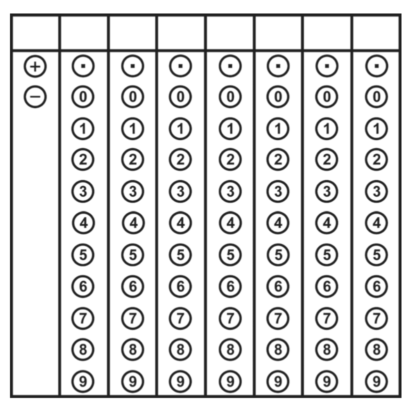
B $\pm 1, \pm 7, \pm 3, \pm\dfrac{3}{7}$

C $\pm 1, \pm\dfrac{8}{3}, \pm\dfrac{11}{3}, \pm\dfrac{7}{3}$

D $\pm 1, \pm 7, \pm\dfrac{1}{3}, \pm\dfrac{7}{3}$

16 What is the value of the geometric series $\displaystyle\sum_{n=0}^{\infty} 48\left(\frac{1}{2}\right)^{n-1}$?

Record the answer and fill in the bubbles on the grid provided. Be sure to use the correct place value.



17 Blake is purchasing a new car for $32,000. If the value of the car decreases at a rate of 9% per year, approximately how many years will it take for the value of the car to reach $15,000?

A 5.90 years

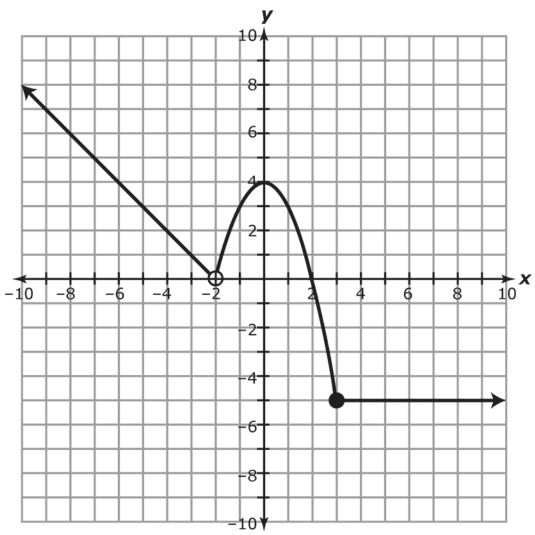
B 6.71 years

C 8.03 years

D 8.79 years

18 Which graph represents $f(x) = \begin{cases} -x - 2 \text{ if } x \leq -2 \\ -(x^2 - 4) \text{ if } -2 \leq x < 3 \\ -6 \text{ if } x \geq 3 \end{cases}$?

A



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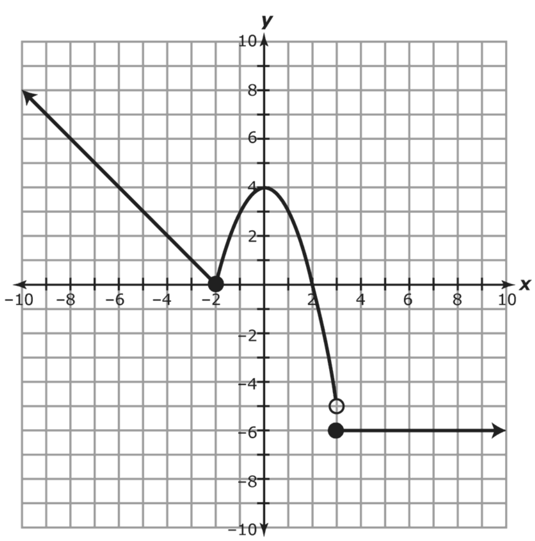

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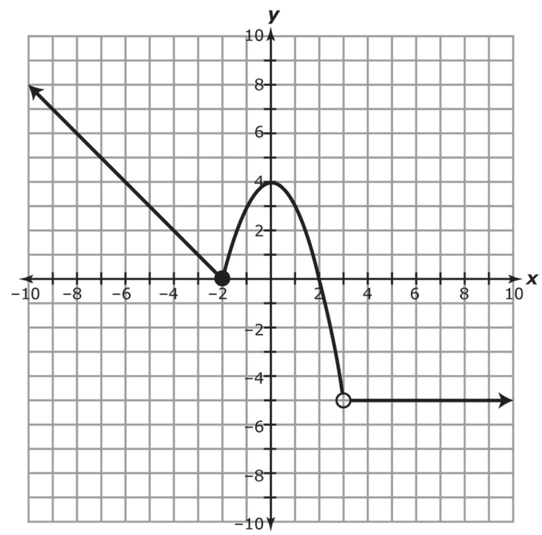

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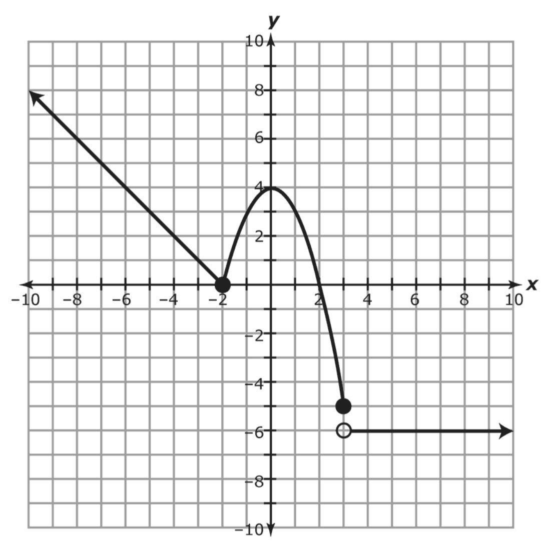
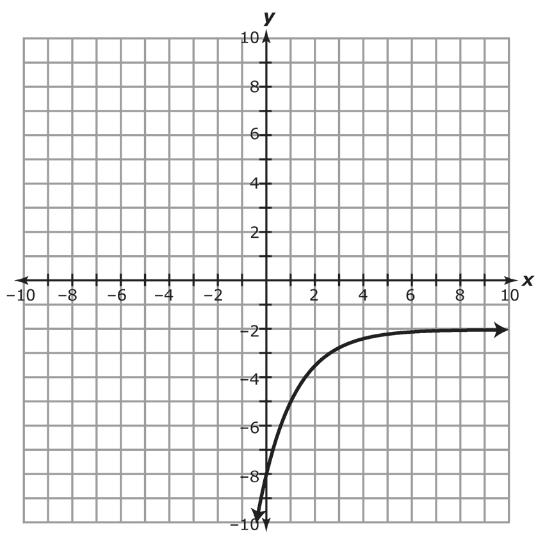


19 The graph of $f(x) = \left(\frac{1}{2}\right)^x$ is transformed using the given steps in the order shown.

- Reflect across the x-axis
- Vertically stretch by a factor of 3
- Translate 1 unit right
- Translate 2 units down

Which graph represents the transformed function?

A



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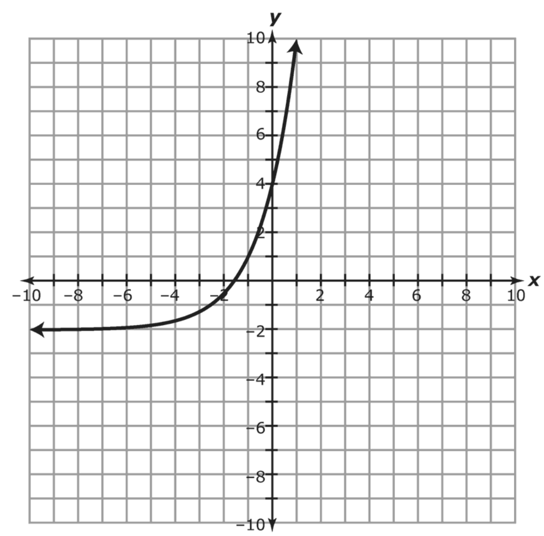

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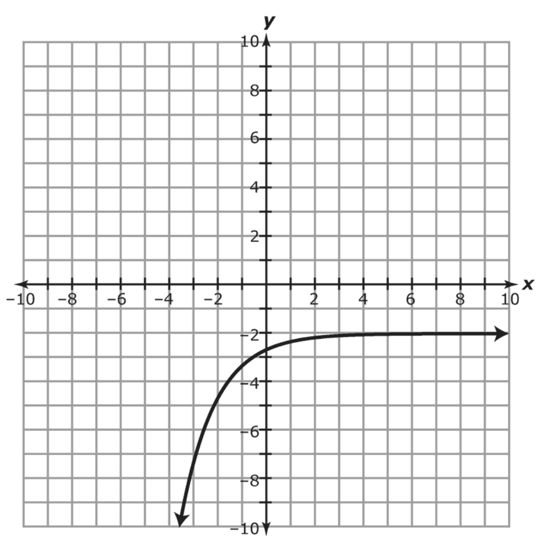

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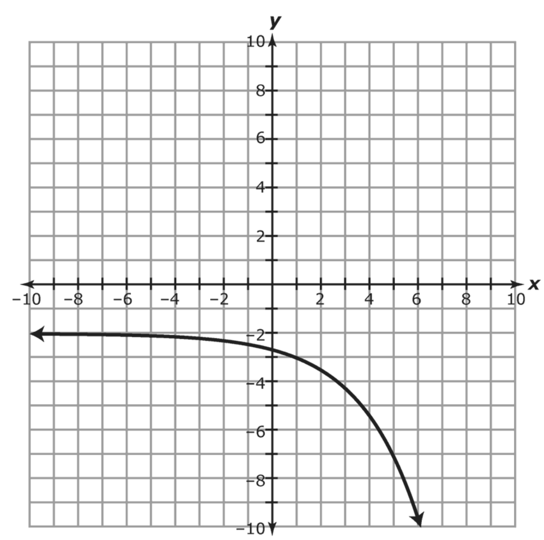


20 What is the domain of the function $f(x) = \dfrac{x}{x^2 + 3x}$?

A $(-\infty, -3) \cup (-3, \infty)$

B $(-\infty, -3] \cup [-3, \infty)$

C $(-\infty, -3] \cup [-3, 0) \cup (0, \infty)$

D $(-\infty, -3) \cup (-3, 0) \cup (0, \infty)$

21 Polly was given a rational function.

$$f(x) = \frac{x+3}{x^2 + x - 6} + 4$$

What discontinuities did Polly discover upon her investigation of this function?

A The graph of $f(x)$ has an infinite discontinuity at $x = 2$ and a removable discontinuity at $x = -3$.

B The graph of $f(x)$ has an infinite discontinuity at $x = -3$ and a removable discontinuity at $x = 2$.

C The graph of $f(x)$ has an infinite discontinuity at $x = 4$ and a removable discontinuity at $x = 3$.

D The graph of $f(x)$ has an infinite discontinuity at $x = 0$.

22 The annual growth rate for an investment is found using the function

$$r = \frac{1}{t} \ln \frac{P}{P_0}$$

where r is the annual growth rate, t is the time in years, P_0 is the initial investment, and P is the present value. Five years ago, Helen invested $5,000 at an annual growth rate of 3.7%. What is the present value of Helen's investment?

A $5,093.36

B $5,188.47

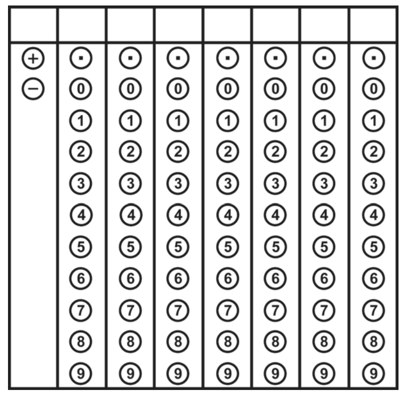
C $6,016.09

D $31,799.10

23 The first three terms of a sequence are shown.

$$-83, -79, -75, \ldots$$

What is the sum of the first 51 terms of this sequence?



Record the answer and fill in the bubbles on the grid provided. Be sure to use the correct place value.

24 The number of downloads of a new song grows at a continuous exponential rate during the first week after the song's release, as shown in the table.

Days, *t*	0	5
Number of downloads, *N(t)*	1,200	9,900

Which function is used to find $N(t)$, the number of downloads after t days?

A $N(t) = 1{,}200(1.5251)^t$

B $N(t) = 1{,}200(1.4220)^t$

C $N(t) = 1{,}200e^{.5251t}$

D $N(t) = 1{,}200e^{.4220t}$

Answer Key

1. A
2. C
3. D
4. C
5. B
6. C
7. B
8. A
9. 40.4
10. D
11. B
12. A
13. C
14. B
15. D
16. 192
17. C
18. B
19. A
20. D
21. A
22. C
23. 867
24. D