AB Calc AP Exam

Name: ________________ School: __________________ Score:

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
11.	12.	13.	14.	15.	16.	17.	18.	19.	20.
21.	22.	23.	24.	25.	26.	27.	28.	29.	30.
31.	32.	33.	34.	35.	36.	37.	38.	39.	40.
41.	42.	43.	44.	45.					

1.

AB Calc AP Exam

Name: ___________________ School: _____________________ Score:

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AB Calc AP Exam

Name: __________________ School: ____________________ Score:

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CALCULUS AB

SECTION I, Part A

Time—60 Minutes

Number of questions—30

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. On what interval(s) is the graph of $y = x - x^3$ concave up?

(A) $(-\infty, 0)$

(B) $(0, \infty)$

(C) $\left(-\infty, -\sqrt{\frac{1}{3}}\right) \cup \left(\sqrt{\frac{1}{3}}, \infty\right)$

(D) $\left(-\sqrt{\frac{1}{3}}, \sqrt{\frac{1}{3}}\right)$

---

2. The Intermediate Value Theorem guarantees a value c, such that $f(c) = 0$ for $f(x) = x^3 + x - 3$ on which of the following intervals?

(A) $(-1, 0)$
(B) $(0, 1)$
(C) $(1, 2)$
(D) $(2, 3)$

---

3. $\lim_{x \to 2^-} \frac{9}{x-2} =$

(A) $-\infty$
(B) 0
(C) 9
(D) The limit does not exist.

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4. Evaluate $\int \sin^3 x \cos x dx$.

(A) $\frac{\sin^2 x}{2} + C$
(B) $\frac{\sin^3 x}{3} + C$
(C) $\frac{\sin^4 x \cos^2 x}{8} + C$
(D) $\frac{\sin^4 x}{4} + C$

---

5. Find the average value of $f(x) = 2e^{2x}$ on the interval [1, 5].

(A) $\frac{1}{2}\left(e^{10} - e^2\right)$
(B) $\frac{1}{4}\left(e^{10} - e^2\right)$
(C) $\frac{1}{4}\left(e^8\right)$
(D) $\frac{1}{2}\left(e^8\right)$

GO ON TO THE NEXT PAGE.

6. A box with a square base, rectangular sides, and no top is to have a volume of 108 feet. What are the dimensions of the rectangular sides that maximize the box's surface area?

(A) 6 feet by 12 feet
(B) 12 feet by 12 feet
(C) 12 feet by 3 feet
(D) 6 feet by 3 feet

---

7. Given $f(x)$ below, for what values of a and b is $f(x)$ differentiable for all values of x ?

$$f(x)=\begin{cases} x^{\frac{5}{3}}+b;\ x<1 \\ ax^{\frac{4}{3}};\ x\geq 1 \end{cases}$$

(A) $a=\frac{5}{4}$ and $b=\frac{1}{4}$
(B) $a=\frac{5}{4}$ and $b=\frac{9}{4}$
(C) $a=\frac{4}{5}$ and $b=-\frac{1}{5}$
(D) $a=\frac{4}{5}$ and $b=\frac{9}{5}$

---

8. Use linear approximation to estimate $\sqrt{35.9}$.

(A) $\frac{59}{10}$
(B) $\frac{61}{10}$
(C) $\frac{719}{120}$
(D) $\frac{721}{120}$

---

9. Evaluate $\int x\left(\sqrt[3]{x+8}\right)dx$.

(A) $\frac{3}{4}(x+8)^{\frac{4}{3}}+C$

(B) $\frac{4}{3}(x+8)^{\frac{4}{3}}+C$

(C) $\frac{3}{7}(x+8)^{\frac{7}{3}}-6(x+8)^{\frac{4}{3}}+C$

(D) $\frac{3}{7}(x+8)^{\frac{7}{3}}+6(x+8)^{\frac{4}{3}}+C$

---

10. Find $\frac{dy}{dx}$ if $y=\frac{1-\tan\left(x^2\right)}{1+\tan\left(x^2\right)}$.

(A) $\frac{4x\sec^2\left(x^2\right)}{\left(1+\tan\left(x^2\right)\right)^2}$

(B) $-\frac{4x\sec^2\left(x^2\right)}{\left(1+\tan\left(x^2\right)\right)^2}$

(C) $\frac{\sec^2\left(x^2\right)}{\left(1+\tan\left(x^2\right)\right)^2}$

(D) $-\frac{\sec^2\left(x^2\right)}{\left(1+\tan\left(x^2\right)\right)^2}$

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11. A shoe manufacturer's profit can be found by the function $P(x) = -x^3 + 6x^2 + 125$, $(x > 0)$, where x is the number of shoes sold (in thousands). The maximum profit for the manufacturer will be when it sells how many shoes?

(A) 0
(B) 1000
(C) 4000
(D) 12,000

---

12. Find $f'(2)$ if $f(x) = x^2 e^{x^2}$.

(A) $4e^4$
(B) $8e^4$
(C) $20e^4$
(D) $32e^4$

---

13. The velocity of a particle at certain times is given in the table below. Approximate the total distance traveled on the interval [0, 3].

Time (*hours*)	Velocity (*mph*)
0	12
1	18
2	22
3	24

(A) 19
(B) 20
(C) 29
(D) 58

---

14. What value of k makes $f(x)$ continuous for all values of x ?

$$f(x)=\begin{cases}-x^2+2kx+66; & x>3\\ 4kx+x^3; & x\leq 3\end{cases}$$

(A) 0
(B) 3
(C) 5
(D) There is no value of k for which $f(x)$ is continuous for all values of x.

---

15. $\lim\limits_{x\to 8}\dfrac{x^2-6x-16}{x^2-2x-48}=$

(A) 0

(B) $\frac{1}{2}$

(C) $\frac{5}{7}$

(D) The limit does not exist.

---

16. Find $\dfrac{d}{dx}\left[f^{-1}(2)\right]$ if $f(x)=\dfrac{x^3-17}{5}$.

(A) $\frac{5}{27}$

(B) 1

(C) $\frac{12}{5}$

(D) $\frac{27}{2}$

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17. Find the absolute maximum of $y = 6x^4 - 3x^2 - 5$ on the interval $[-1, 2]$.

(A) -5

(B) $\frac{43}{8}$

(C) 2

(D) 79

18. Find $\frac{dy}{dx}$ if $y = (\sec(3x))(\ln(5x))$.

(A) $(\sec(3x))\left(\frac{5}{x}\right) + (\sec(3x)\tan(3x))(\ln(5x))$

(B) $(\sec(3x))\left(\frac{1}{5x}\right) + 3(\sec(3x)\tan(3x))(\ln(5x))$

(C) $(\sec(3x))\left(\frac{5}{x}\right) + 3(\sec(3x)\tan(3x))(\ln(5x))$

(D) $(\sec(3x))\left(\frac{1}{x}\right) + 3(\sec(3x)\tan(3x))(\ln(5x))$

19. $\lim_{\theta \to 0} \frac{\tan 7\theta}{\theta} =$

(A) 0
(B) 1
(C) 7
(D) The limit does not exist.

GO ON TO THE NEXT PAGE.

20. Find $\frac{dy}{dx}$ if $y = \cos^5(1 - x^2)$.

(A) $10x\cos^4(1 - x^2)\sin(1 - x^2)$
(B) $-10x\cos^4(1 - x^2)\sin(1 - x^2)$
(C) $5x\cos^4(1 - x^2)$
(D) $10x\cos^4(1 - x^2)$

---

21. Find $\frac{dy}{dx}$ if $y = \frac{5x^4 - 3x^3 + x^2}{x^3}$.

(A) $\frac{20x^3 - 9x^2 + 2x}{x^6}$

(B) $5 + \frac{1}{x^2}$

(C) $\frac{20x^3 - 9x^2 + 2x}{3x^2}$

(D) $5 - \frac{1}{x^2}$

---

22. Find $\frac{dy}{dx}$ if $y = \arcsin\left(4\sqrt{x}\right)$.

(A) $\frac{2}{x - 16x^2}$

(B) $\frac{1}{4\sqrt{x - 16x^2}}$

(C) $\frac{4}{x - x^2}$

(D) $\frac{2}{\sqrt{x - 16x^2}}$

23. Evaluate $\lim_{x \to 0} \frac{x2^x}{2^x - 1}$.

(A) 0

(B) $\frac{1}{\ln 2}$

(C) $\frac{1}{2}$

(D) The limit does not exist.

24. Find $\frac{dy}{dx}$ at the point (2, 1) if $3x^2 + 2xy^2 - 3y^2 = 13$.

(A) $\frac{dy}{dx} = -14$

(B) $\frac{dy}{dx} = -7$

(C) $\frac{dy}{dx} = 7$

(D) $\frac{dy}{dx} = 14$

25. Find $y(-2)$ if $\frac{dy}{dt} = -4y$ and $y(0) = 12$.

(A) 12
(B) $12e^{-8}$
(C) $12e^{8}$
(D) $12e^{16}$

GO ON TO THE NEXT PAGE.

26. Find $\frac{dy}{dx}$ if $y = \frac{\sin 2x}{\sin x} - \frac{\cos 2x}{\cos x}$.

(A) $\sec x \tan x$
(B) $2\sec x \tan x$
(C) $4\sec^2 x$
(D) $4\sec^2 x \tan x$

---

27. If a spherical balloon's radius is increasing at 12 inches per second, how fast is the volume of the balloon increasing when the radius is 2 inches?

(A) 12π cubic inches per second
(B) 48π cubic inches per second
(C) 192π cubic inches per second
(D) 256π cubic inches per second

---

28. An object's height above the ground is given by the equation $y(t) = -3t^2 + 24t + 12$, $t \geq 0$, and its horizontal location is given by the equation $x(t) = -2t + 16$. What is its horizontal location when the object reaches its maximum height?

(A) 0
(B) 8
(C) 12
(D) 16

---

29. Which of the following is a solution to the differential equation $\frac{d^2y}{dx^2} = 4y$?

(A) $y = e^{2x}$

(B) $y = \sin 2x$

(C) $y = \cos 2x$

(D) $y = x^4$

---

30. If the position of a particle is given by the function $s(t) = 2t^3 - 21t^2 + 60t - 42$, $t \geq 0$, for what value(s) of t is the particle changing direction?

(A) $t = 2$
(B) $t = 5$
(C) $t = 2, 5$
(D) The particle does not change direction.

END OF PART A, SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS AB

SECTION I, Part B

Time—45 Minutes

Number of questions—15

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test:

1. The **exact** numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
2. Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

31. Find $\frac{d}{dx}\int_5^{\sqrt{x}} \cos\left(t^2\right) dt$.

(A) $\frac{\cos x}{2\sqrt{x}}$

(B) $\cos x$

(C) $-\sin x$

(D) $-\frac{\sin x}{2\sqrt{x}}$

32. If $g(x)=\int_0^x (t^3-t^2+1)\,dt$, find $g(2)$.

(A) 4

(B) 5

(C) $\frac{10}{3}$

(D) $\frac{13}{3}$

---

33. Evaluate $\int_0^1 \frac{12dx}{1+36x^2}$.

(A) arctan 6
(B) 2arctan 6
(C) 6arctan 6
(D) 12arctan 6

---

34. If $\int_{-2}^{2} f(x)dx = 8$, $\int_{2}^{5} f(x)dx = -16$, and $\int_{10}^{5} f(x)dx = 18$, what is $\int_{-2}^{10} f(x)dx$?

(A) –26
(B) 6
(C) 10
(D) 42

---

35. On what interval(s) is $f = 2x\cos x$ increasing on $[0, \pi]$?

(A) $(1.1571, \pi)$
(B) $(0, 1.571)$
(C) $(0, 0.860)$
(D) $(0.860, \pi)$

---

36. Evaluate $\lim\limits_{h \to 0} \dfrac{(3+h)^4 - 81}{h}$.

(A) 0
(B) 81
(C) 108
(D) The limit does not exist.

---

37. If a particle's acceleration is given by $a(t) = t - 2$, $t \geq 0$, with its initial velocity $v(0) = 12$ and initial position $s(0) = 40$, find the position equation $s(t)$.

(A) $s(t) = t^2 - 2t + 12$

(B) $s(t) = \dfrac{t^2}{2} - 2t + 12$

(C) $s(t) = \dfrac{t^3}{6} - t^2 + 12t + 40$

(D) $s(t) = \dfrac{t^3}{2} - t^2 + 12t + 40$

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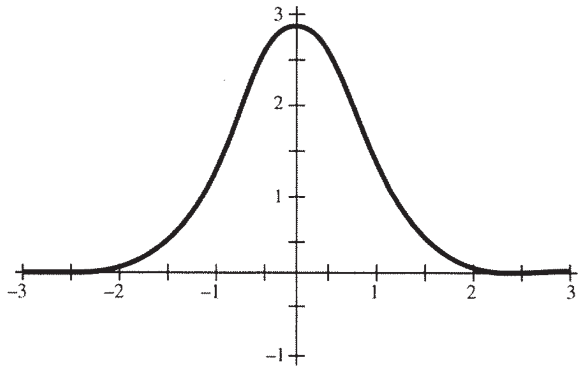
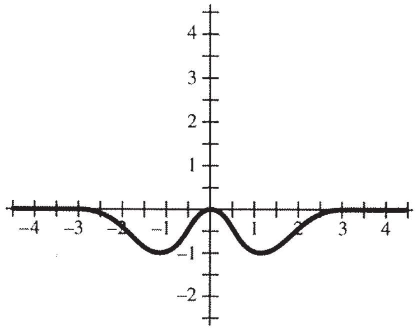
38. Which of the following is the graph of $f'(x)$ if the graph of $f(x)$ is



(A)



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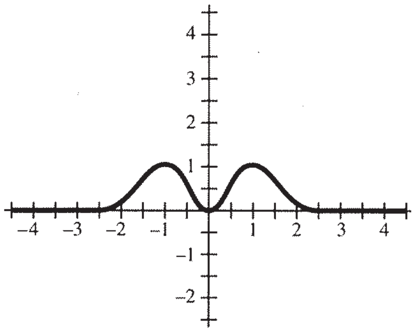

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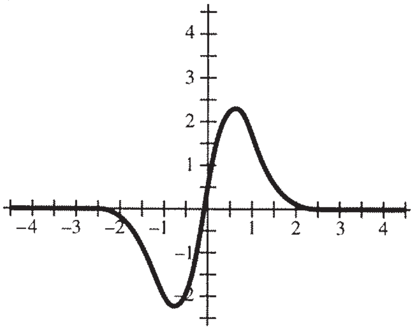

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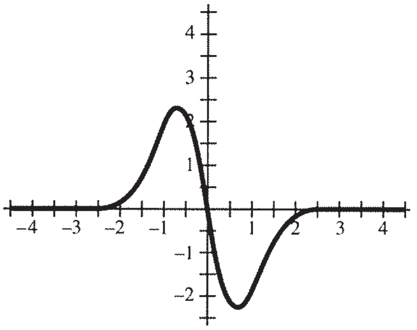

39. Evaluate $\int \frac{\sin x}{3-\cos x}dx$.

(A) $-\ln|3-\cos x|+C$

(B) $\ln|3-\cos x|+C$

(C) $\frac{(3-\cos x)^2}{2}+C$

(D) $-\frac{(3-\cos x)^2}{2}+C$

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40. The table below gives values for $f(x)$ and $g(x)$, and their derivatives, for certain values of x. If $h(x)=f(x^2)\,g(x)$, find $h'(2)$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
–2	8	3	0	1
–1	6	1	3	12
2	5	6	–2	24
4	7	–1	5	4
16	0	2	1	–24

(A) 120
(B) 198
(C) 288
(D) 328

---

41. Find the area between $y = \sin x$ and $y = \frac{1}{2}x$ on the interval $[0, \pi]$.

(A) -1.309
(B) -0.467
(C) 1.309
(D) 1.574

---

42. If the velocity of a particle in meters per second is given by $v(t) = t^2 - 7t + 10$, $t \geq 0$, find the distance that the particle travels in the time interval $[0, 3]$.

(A) 7
(B) 9.833
(C) 10.666
(D) 16

---

43. Find y if $\frac{dy}{dx} = \frac{1}{(x-1)^2}$ and $y(0) = 10$.

(A) $y = \frac{1}{(x-1)^3} + 10$

(B) $y = \frac{1}{(x-1)^3} + 9$

(C) $y = -\frac{1}{x-1} + 10$

(D) $y = -\frac{1}{x-1} + 9$

---

toward derivative

44. Find the value of c that is guaranteed by the Mean Value Theorem for $f(x) = x + \frac{1}{x}$ on the interval [1, 3].

(A) 1.414
(B) 1.155
(C) 1.732
(D) There is no value of c.

---

45. Approximate the area between the parabola $y = 6x - x^2$ and the x-axis using four right-hand rectangles on the interval [0, 6].

(A) 9
(B) 23.625
(C) 33.75
(D) 36

STOP

END OF PART B, SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART B ONLY.

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.

SECTION II
GENERAL INSTRUCTIONS

You may wish to look over the problems before starting to work on them, since it is not expected that everyone will be able to complete all parts of all problems. All problems are given equal weight, but the parts of a particular problem are not necessarily given equal weight.

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS ON THIS SECTION OF THE EXAMINATION.

- You should write all work for each part of each problem in the space provided for that part in the booklet. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed-out work will not be graded.
- Show all your work. You will be graded on the correctness and completeness of your methods as well as your answers. Correct answers without supporting work may not receive credit.
- Justifications require that you give mathematical (non-calculator) reasons and that you clearly identify functions, graphs, tables, or other objects you use.
- You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt (X^2, X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

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SECTION II, PART A

Time—30 minutes

Number of problems—2

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS

During the timed portion for Part A, you may work only on the problems in Part A.

On Part A, you are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

1. Oil is being pumped into a cavern for storage. The cavern is 400 meters deep. The area of the horizontal cross section of the chamber at a depth y is given by the function A, where $A(y)$ is measured in square meters. The function A is continuous and decreases as depth increases. Selected values for $A(y)$ are given in the table below.

y (meters)	0	100	150	250	400
$A(y)$ (square meters)	65	38.4	30.6	20.2	11.3

(a) Use a right Riemann sum with the four subintervals indicated by the data in the table to approximate the volume of the chamber. Indicate the units of measure.

(b) Does the approximation in part (a) overestimate or underestimate the volume of the cavern? Explain your reasoning.

(c) The area in square meters of the horizontal cross section at depth y is modelled by the function f given by

$f(y) = \dfrac{65}{e^{0.004y} + .002y}$. Based on this model, find the volume of the tank. Indicate the units of measure.

(d) Oil is pumped into the cavern. When the depth of the oil is 200 meters, the depth is increasing at a rate of 0.45 meter per minute. Using the model from part (c), find the rate at which the volume of oil is changing with respect to time when the depth of the oil is 200 meters. Indicate the units of measure.

GO ON TO THE NEXT PAGE.

2. When a subway leaves its initial station, there are 25 passengers on board. Passengers leave a subway at a rate modelled by the function $f(t)$ given by

$$f(t) = 12 + (0.5t)\sin\left(\frac{t^2}{40}\right); 0 \le t \le 60$$

where $f(t)$ is measured in passengers per minute and t is measured in minutes since the subway leaves its initial station. After the subway has been traveling for 20 minutes, passengers board the subway at a rate modelled by

$$g(t) = 20 + 3.2\ln(t^2 + 4t); 10 \le t \le 60$$

where the function $g(t)$ is measured in passengers per minute and t is the number of minutes since the subway leaves its initial station.

(a) How many passengers leave the subway during the time interval $0 \le t \le 20$? Round to the nearest passenger.

(b) Find $f'(20)$. Using the correct units, explain the meaning of $f'(20)$ in the context of this problem.

(c) Is the number of passengers on the subway increasing or decreasing at time $t = 20$? Justify your reason for your answer.

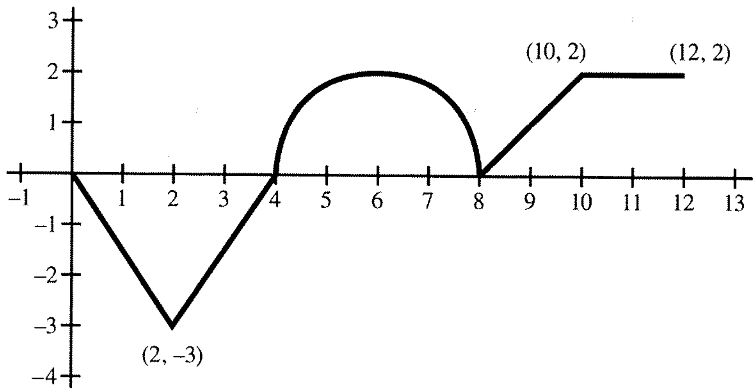
(d) How many passengers are on the subway at time $t = 30$? Round to the nearest passenger.

SECTION II, PART B
Time—1 hour
Number of problems—4

NO CALCULATOR IS ALLOWED FOR THESE PROBLEMS

During the timed portion for Part B, you may continue to work on the problems in Part A without the use of any calculator.

3.



Graph of $f'(x)$

The function f is differentiable on the closed interval [0, 12] and $f(0) = 7$. The graph of $f'(x)$ above consists of a semicircle and four line segments.

(a) Find the values of $f(4)$ and $f(12)$.

(b) On which intervals is f decreasing? Justify your answer.

(c) Find the absolute minimum value of f on the closed interval [0, 12]. Justify your answer.

(d) Find $f''(1)$.

4. At time $t = 0$, an iron ingot is taken from a furnace and placed on an anvil to cool. The internal temperature of the ingot is 286 degrees Celsius (°C) at time $t = 0$ and the internal temperature is greater than room temperature, 30 degrees Celsius. The internal temperature of the ingot at time t minutes can be modelled by the function I that satisfies the differential equation $\frac{dI}{dt} = -\frac{1}{32}(I - 30)$, where $I(t)$ is measured in degrees Celsius and $I(0) = 286$.

(a) Write an equation for the line tangent to the graph of I at $t = 0$. Use this equation to approximate the internal temperature of the ingot at time $t = 0$.

(b) Use $\frac{d^2I}{dt^2}$ to determine whether your answer in part (a) is an overestimate or an underestimate of the internal temperature of the ingot at time $t = 8$.

(c) If we immerse the ingot in a cooling bath, another model of the internal temperature of the ingot at time t minutes is the function B that satisfies the differential equation $\frac{dB}{dt} = -(B - 30)^{\frac{3}{4}}$, where $B(t)$ is measured in degrees Celsius and $B(0) = 286$. Using this model, what is the internal temperature of the ingot at time $t = 8$?

GO ON TO THE NEXT PAGE.

5. Consider the curve given by $2xy - y^2 = 3$.

(a) Write an equation for the tangent line to the curve at the point (2, 1).

(b) Find the coordinates of all points on the curve at which the line tangent to the curve at that point is vertical.

(c) Evaluate $\frac{d^2y}{dx^2}$ at the points on the curve where $x = 2$ and $y = 1$.

6. Two particles move along the x-axis. For $0 \le t \le 10$, the velocity of particle A at time t is given by $v_A(t) = t^2 - 5t + 4$ and the position of particle B is at time t is given by $x_B(t) = \frac{t^3}{3} - 4t^2 + 12t$. Particle A is at position $x = 3$ at time $t = 0$.

(a) For $0 \le t \le 10$, when is particle B moving to the left?

(b) For $0 \le t \le 10$, find all times t when the two particles are traveling in the same direction.

(c) Find the acceleration of particle A at time $t = 3$. Is its speed increasing, decreasing, or neither at time $t = 3$? Justify your answer.

(d) Find the position of particle A the first time that it changes direction.

STOP

END OF EXAM

T1

1. A	2. C	3. A	4. D	5. B	6. D	7. A	8. C	9. C	10. B
11. C	12. C	13. D	14. C	15. C	16. A	17. D	18. D	19. C	20. A
21. D	22. D	23. B	24. B	25. C	26. A	27. C	28. B	29. A	30. C
31. A	32. C	33. B	34. A	35. C	36. C	37. C	38. D	39. B	40. C
41. C	42. B	43. D	44. C	45. C					