Math Power

November 30, 2018

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Name: (First)____________(Last)____________

School: _________________ Grade:______

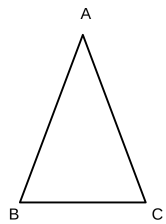
Geom

Isosceles Triangle

THEOREM A

In ΔABC, if ∠B = ∠C, then

AB = AC.



Proof:

Choose AD as the angle bisector of ∠BAC, then

∠1 = ∠2.

Also, we have

∠B = ∠C (given)

∠3 = 180° - (∠1 + ∠B)

= 180° - (∠2 + ∠C)

= ∠4

Since

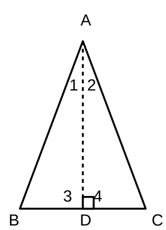
AD = AD (common side),

we have established

ΔADB ≅ ΔADC (ASA)

thus

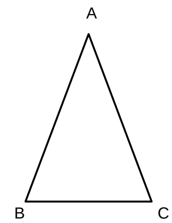
AB = AC.



THEOREM B

In ΔABC, if AB = AC, then

∠B = ∠C.



Proof:

Choose AD as the angle bisector of ∠BAC, then

∠1 = ∠2.

Since

AB = AC (given)

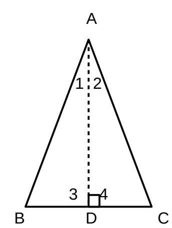
AD = AD (common side)

we have established

ΔADB ≅ ΔADC,

thus

∠B = ∠C.



From now on we may say that a triangle is an isosceles if

it has a pair of congruent sides,

or

it has a pair of congruent angles.

<u>*Question set*</u> [**1 - 2**]

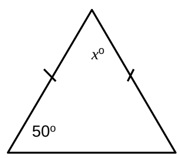
Prove: each of the following statements.

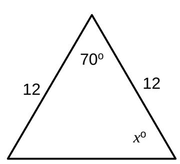
1. An equilateral has three 60° angles.

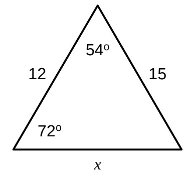
2. An equiangular triangle is also equilateral triangle.

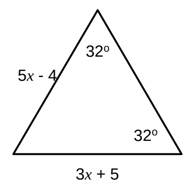
Question set [**3 - 7**]

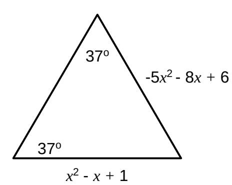
The diagram is not drawn to scale in each of the following problems. Find the value of *x*.

3. 

4. 

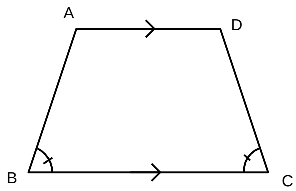
5. 

6. 

7. 

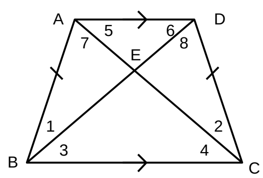
Question set [**8 - 11**]

The diagram below is an isosceles trapezoid, which is a special trapezoid with ∠B = ∠C.



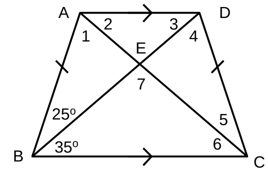
8. Prove: AB = DC.

9. Prove: ∠1 = ∠2, ∠3 = ∠4, ∠5 = ∠6, and ∠7 = ∠8.

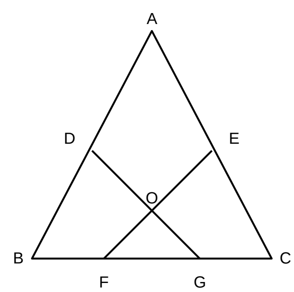


10. Prove: AC = BD.

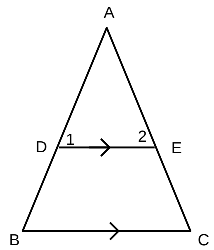
11. ABCD is an isosceles trapezoid. Find all the measures of the angles from ∠1 to ∠7.



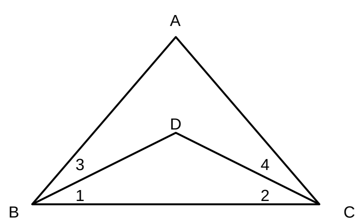
12. ΔABC as below is an isosceles with AC = AB. D is the midpoint of AB, E is the midpoint of AC. DG and EF meets at O with OD = OE.
Prove: OF = OG.



13. Given: DE//BC and ∠1 = ∠2.
    Prove: DB = EC.

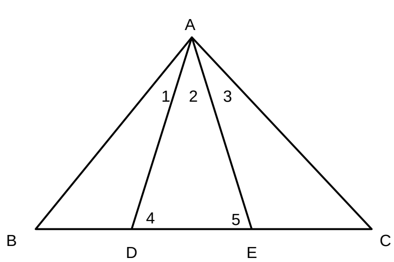


14. Given AB = AC and DB = DC.
    Prove: ∠3 = ∠4.



<u>Question set</u> [**15 - 16**]

Given AB = AC. AD and AE trisect ∠BAC.

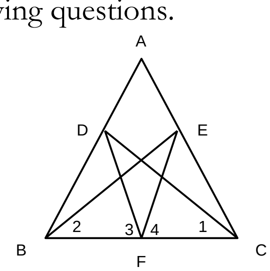


15. How can you say about ∠1, ∠2 and ∠3?

16. Prove: AD = AE.

<u>Question set</u> [**17 - 18**]

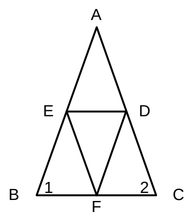
Use the diagram below to answer the following questions.



17. Given AD = AE and BD = CE. Prove: ∠1 = ∠2.

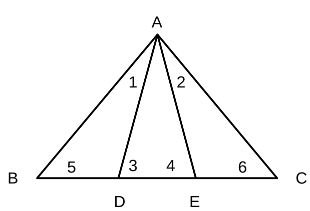
18. Do you think the previous condition is sufficient to prove ∠3 = ∠4? If not, refine it.

19. Given ΔABC is an isosceles triangle with $AB = AC$. Join the three midpoints of the three sides to form a triangle. Prove: the new triangle is also an isosceles.



Question set [**20 - 24**]

ΔABC is depicted as below. If the given congruent conditions are sufficient, then indicate which are congruent pairs in the list
(i) (AB, AC)
(ii)(AD, AE)
(iii) (BD, CE)
(iv) ($\angle 1$, $\angle 2$)
(v) ($\angle 3$, $\angle 4$)
(vi) ($\angle 5$, $\angle 6$)
Also, briefly state the strategy of your proof. Otherwise, indicate it insufficient.



20. $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$.

21. $BD = EC$ and $\angle 3 = \angle 4$

22. $\angle 1 = \angle 2$

23. $AB = AC$ and $\angle 3 = \angle 4$.

24. $AD = AE$ and $\angle 1 = \angle 2$

Right Triangle

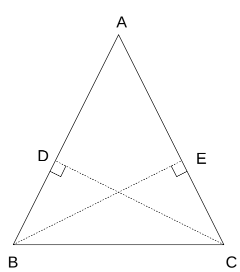
THEOREM C
[Right triangles, a special case]
Two right triangles are congruent if both fulfill any of the following:
LL: congruent leg-leg
HL: hypotenuse-leg
AS: congruent side and an acute angle.

Proof:
Using Pythagorean theorem, we have know the third side can be derived given two sides. Therefore, LL and HL is a special case of SSS. AS is again a special case of AAS of ASA since both have a right angle and an acute angle, the third angle can be also derived.

25. Prove: a rectangle is divided into two congruent right triangles by one diagonal.

<u>*Question set*</u> [**26 - 27**]

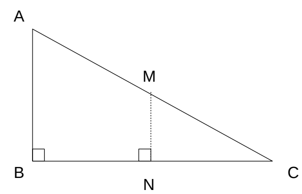
CD⊥AB, and BE⊥AC.



26. Given AB = AC.
    Prove BE = CD.

27. Given that BE = CD.
    Prove: AB = AC.

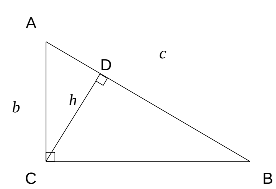
28. ΔABC is a right triangle as depicted. MN is the perpendicular bisector of BC, prove that BM is a median, namely,
    AM = MC.



THEOREM D
[Pythagorean Theorem]
ΔABC is a right triangle with sides as specified. Then, there is a relationship among the sides:

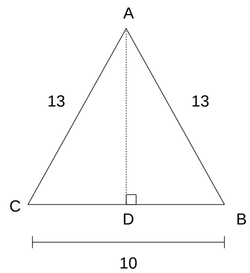
$$c^2 = a^2 + b^2$$

Moreover, $h = \frac{ab}{c}$



Question set [**29 - 31**]

ΔABC is an isosceles triangle as specified.
Find the area.



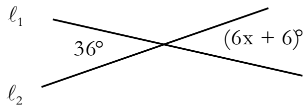
29. Find the height AD.

30. Find the area of ΔABC.

31. Find the height from B to AC.

Assessment Test

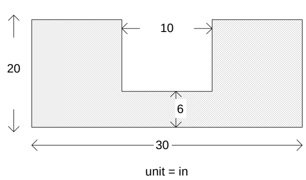
32. What is the value of x in the figure below?



33. When a certain rectangle is divided into half, two squares are formed. If each of these squares has perimeter 48, what is the perimeter of the original rectangle?

34. You'd like to have a garden with area 18 ft^2. You have a space 6 ft long in which to put the garden. How long does the fencing require?

Question set [**35 - 36**]

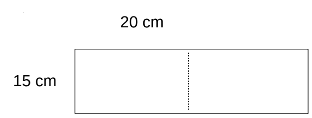


35. What is the area of the shape?

36. What is the perimeter of the shape?

Question set [**37 - 38**]

Two identical rectangles are combined to make a larger rectangle.

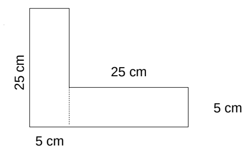


37. What is the perimeter?

38. What is the area?

Question set [**39 - 40**]

Two identical rectangles with width = 5 and length = 25 are connected form an "L" shape.

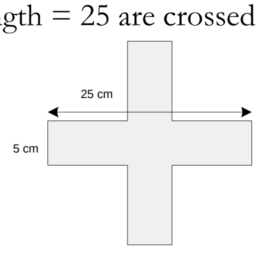


39. What is the area of the combined shape?

40. What is the perimeter of the combined shape?

Question set [**41 - 42**]

Two identical rectangles with width = 5 and length = 25 are crossed to form a cross.

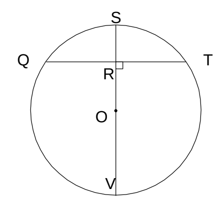


41. What is the area of the figure?

42. What is the perimeter of the figure?

Question set [**43 - 45**]

Straight line SV is a diameter of circle O. QT ⊥SV.



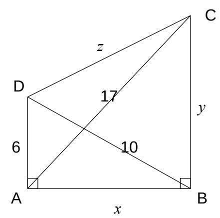
43. If QT = 10 and OR = 12, then the radius =

44. If OR = 4 and RS = 4, then QT =

45. If QT = 12 and SR = 3, then the radius =

Question set [**46 - 48**]

The dimensions of trapezoid ABCD are depicted. Find the values of the following. (Hint: This problem requires using Pythagorean theorem repeatedly.)



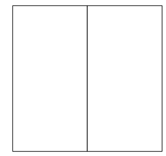
46. $x =$

47. $y =$

48. $z =$

Question set [**49 - 51**]

Two rectangles form a square. The square has an area of 64 in^2.



49. What is the area of a rectangle?

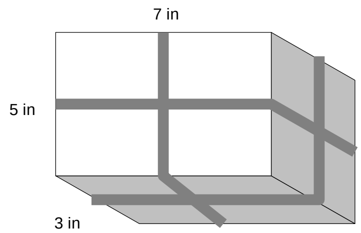
50. What is the length of a rectangle?

51. What is the perimeter of a rectangle?

Assessment Test

Question set [**52 - 53**]

A box is enclosed by wrapping paper. Ribbon is then strapped around the box. Assume there is no overlapping of wrapping paper; neither a knot or bow of the ribbon.



52. Find the area of the wrapping paper.
    (A) 130 in^2
    (B) 142 in^2
    (C) 150 in^2
    (D) 162 in^2

53. Find the length of the ribbon.
    (A) 60 in
    (B) 72 in
    (C) 80 in
    (D) 92 in

Question set [**54 - 56**]

2-D Metric: area/surface area formulae review.

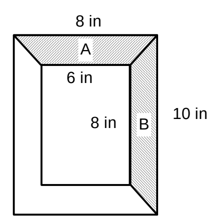
54. Charlie wants to know the area of his property, which measures 120 ft by 150 ft. Which formula will he use?
    (A) A = s^2
    (B) A = πr^2
    (C) A = $\frac{1}{2}hb$
    (D) A = lw

55. Cathy is creating a quilt out of fabric panel that is 6 in by 6 in. She wants to know the area of her quilt. Which formula will she use?
    (A) A = s^2
    (B) A = $\frac{1}{2}bh$
    (C) A = πr^2
    (D) A = $\frac{1}{2}h(b_1 + b_2)$

56. Rick is ordering a new triangular sail for his boat. He needs to know the area of the sail. Which formula will he use?
    (A) A = lw
    (B) A = $\frac{1}{2}bh$
    (C) A = bh
    (D) A = $\frac{1}{2}h(b_1 + b_2)$

Question set [**57 - 59**]

The picture frame shown below has outer dimensions of 8 in by 10 in and inner dimensions of 6 in by 8 in.



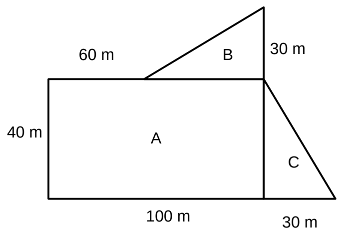
57. Find the area of section A of the frame.
    (A) 18 in^2
    (B) 14 in^2
    (C) 7 in^2
    (D) 9 in^2

58. Find the area of section B of the frame.
    (A) 18 in^2
    (B) 14 in^2
    (C) 7 in^2
    (D) 9 in^2

59. Find the area of the frame.
   (A) 18 in^2
   (B) 14 in^2
   (C) 32 in^2
   (D) 36 in^2

Question set [**60 - 61**]

John is planning to purchase an irregularly shaped plot of land.



60. Find the total area of the land.
   (A) 6,400 m^2
   (B) 5,200 m^2
   (C) 4,500 m^2
   (D) 4,600 m^2

61. Determine the perimeter of the plot of land.
   (A) 260 m
   (B) 340 m
   (C) 360 m
   (D) 320 m

62. A racquetball court is 40 ft by 20 ft. What is the area of the court in square feet?
   (A) 60 ft^2
   (B) 80 ft^2
   (C) 800 ft^2
   (D) 120 ft^2

63. Allan has been hired to mow the school soccer field, which is 180 ft wide by 330 ft long. If his mower mows strips that are 2 feet wide, how many times must he mow across the width of the lawn?
   (A) 90
   (B) 165
   (C) 255
   (D) 60

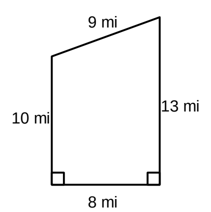
64. Erin is painting a bathroom with four walls each measuring 8 ft by 5.5 ft. Ignoring the doors or windows, what is the area to be painted?
   (A) 176 ft^2
   (B) 88 ft^2
   (C) 54 ft^2
   (D) 160 ft^2

65. Karen is buying a wallpaper border for her bedroom, which is 12 ft by 13 ft. If the border is sold in rolls of 5 yards each, how many rolls will she need to purchase?
   (A) 3
   (B) 4
   (C) 5
   (D) 6

66. Brittney would like to carpet her bedroom. If her room is 11 ft by 13 ft, what is the area to be carpeted in square feet?
   (A) 121 ft^2
   (B) 48 ft^2
   (C) 169 ft^2
   (D) 143 ft^2

67. If a triangular sail has a vertical height of 83 ft and horizontal length of 30 ft, what is the area of the sail?
(A) 1,245 ft^2
(B) 1,155 ft^2
(C) 201 ft^2
(D) 2,490 ft^2

68. Stuckeyburg is a small town in rural America. Use the map to approximate the area of the town.



(A) 40 $miles^2$
(B) 104 $miles^2$
(C) 93.5 $miles^2$
(D) 92 $miles^2$

69. Mark intends to tile a kitchen floor, which is 9 ft by 11 ft. How many 6-inch tiles are needed to tile the floor?
(A) 60
(B) 99
(C) 396
(D) 449

70. A cable is attached to a pole 24 ft above ground and fastened to a stake 10 ft from the base of the pole. In order to keep the pole perpendicular to the ground, how long is the cable?
(A) 22 ft
(B) 26 ft
(C) 20 ft
(D) 18 ft

71. Mark is constructing a walkway around his inground pool. The pool is 20 ft by 40 ft and the walkway is intended to by 4 ft wide. What is the area of the walkway?
(A) 224 ft^2
(B) 416 ft^2
(C) 256 ft^2
(D) 544 ft^2

Answer Key

Isosceles Triangle

1. $\frac{1}{3}\times 180° = 60°$

2. It follows from the previous the statement.

3. 80

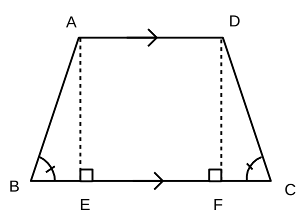
4. 55

5. 12

6. 4.5
   $5x - 4 = 3x + 5$
   $2x = 9$
   $x = 4.5$

7. 0.5
   $5x^2 + 8x - 6 = -x^2 + x - 1$
   $6x^2 + 7x - 5 = 0$
   $(3x + 5)(2x - 1) = 0$
   $x = 0.5$ or $-5/3$

8. Draw two auxiliary lines (altitudes) AE and DF.



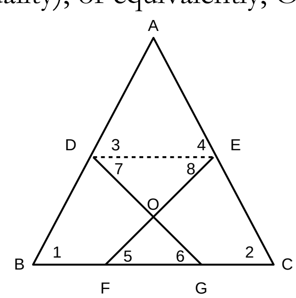
   AE = DF (AEFD is a rectangle, esp. a parallelogram)
   $\angle B = \angle C$
   $\angle AEB = \angle DFC$ (= 90°, altitudes)
   $\Delta ABE \cong \Delta DCF$ (AAS)
   Thus, AB = DC

9. Once you prove that $\Delta ABC \cong \Delta DCB$, then it is obvious that $\angle 3 = \angle 4$, therefore, $\angle 1 = \angle 2$. Similarly, $\angle 7 = \angle 8$ (because $\Delta ABC \cong \Delta DCB$), it follows that $\angle 5 = \angle 6$.

10. 1) AB = CD (proved)
    2) $\angle 1 = \angle 2$ (proved)
    $\angle BAD = 180° - \angle ABC$ (consec. int. AD//BC)
    $\angle CDA = 180° - \angle DCB$ (consec. int. AD//BC)
    3) $\angle BAD = \angle CDA$ (transitivity)
    Combining 1), 2) and 3), we can conclude that $\Delta BAD \cong \Delta CDA$ (AAS). Thus, we have
    AC = BD

11. $\angle 6 = 35°$,
    $\angle 1 = 180° - 60° - 35° = 85° = \angle 4$.
    $\angle 2 = \angle 6 = 35° = \angle 3$ (alternate interior).
    $\angle 5 = 25°$.
    $\angle 7 = 180° - 2\times 35° = 110°$.

12. ΔADE is an isosceles. $\angle 1 = \angle 2 = \angle 3 = \angle 4$. DE//BC (for congruent corresponding angles). Since $\angle 7 = \angle 8$ (for OD = OE), $\angle 5 = \angle 8$ (alternate interior) and $\angle 6 = \angle 7$ (alternate interior). Therefore, $\angle 5 = \angle 6$ (transitivity of equality), or equivalently, OF = OG.



13. First, we claim
    $\angle B = \angle C$,
    since $\angle B$
    $= \angle 1$ (corresponding angle)
    $= \angle 2$ (given)
    $= \angle C$ (corresponding angle)
    Then, we claim
    $AB = AC$ ($\angle B = \angle C$)
    $AD = AE$ ($\angle 1 = \angle 2$)
    Last, we establish
    $BD = EC$
    since $BD = AB - AD = AC - AE = EC$.

14. $\angle ABC = \angle ACB$ ($AB = AC$, given)
    $\angle 1 = \angle 2$ ($DB = DC$, given)
    $\angle 3 + \angle 1 = \angle ABC$
    $\angle 4 + \angle 2 = \angle ACB$
    $\angle 1 = \angle 2$ (cancellation property)

15. $\angle 1 = \angle 2 = \angle 3$

16. $\angle 1 = \angle 3$ (given)
    $\angle B = \angle C$ ($AB = AC$, given)
    $\angle 4 = \angle 1 + \angle B$
    $\angle 5 = \angle 3 + \angle C$
    $\angle 4 = \angle 5$ (transitivity)

17. $\Delta ABE \cong \Delta ACD$ (SAS). So, $\angle ABE = \angle ACD$ and $AB = AC$. Now that ΔABC is an isosceles, $\angle ABC = \angle ACB$. We have $\angle 1 = \angle ACB - \angle ACD = \angle ABC - \angle ABE = \angle 2$.

18. It is not sufficient. The point F must be conditioned properly to have $\angle 3 = \angle 4$. For instance, F is the midpoint of BC, then the statement is true. Use ΔCFD and ΔBFE as container triangles. They are congruent for SAS. Now that $\angle CFD = \angle BFE$, we have $\angle 3 = \angle 4$.

19. 1) $\angle 1 = \angle 2$ (given)
    2) $CF = BF$ (midpoint)
    $AB = AC$ (given)
    $BE = \frac{1}{2}AB$ (midpoint)
    $CD = \frac{1}{2}AC$ (midpoint)
    3) $BE = CD$ (transitivity)
    Combining 1), 2) and 3), we conclude that $\Delta BEF \cong \Delta CDF$ (SAS).
    Therefore, $FE = FD$ and ΔFED is an isosceles triangle.

20. Sufficient, the rest four pairs in the list are congruent since $\Delta ABD \cong \Delta ACE$ for ASA.

21. Sufficient, the rest four pairs in the list are congruent since $\Delta ABD \cong \Delta ACE$ for SAS.

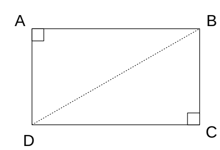
22. Insufficient

23. Sufficient, the rest four pairs in the list are congruent since $\Delta ABD \cong \Delta ACE$ for AAS.

24. Sufficient, the rest of four pairs in the list are congruent since $\Delta ABD \cong \Delta ACE$ for ASA.

Right Triangle

25. $AB = BC$
    $\angle A = \angle C = 90°$
    $AB = CD$
    $\Delta ABD \cong \Delta DCB$.



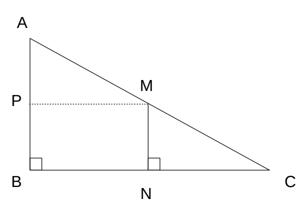
26. Method I:
Since BE is the altitude to side AC, we have
$\text{area}(\Delta ABC) = \frac{1}{2}AC \times BE$
Similarly, we have
$\text{area}(\Delta ABC) = \frac{1}{2}AC \times CD$
Thus, by cancellation we have
$BE = CD$

Method II:
Two right triangles ΔAEB and ΔADC are congruent for AAS (since AB = AC and ∠A is shared).

27. Method I:
CD = BE (given)
$\frac{1}{2}AB \times AD = \frac{1}{2}AC \times BE$ (the area)
AB = AC (cancellation)

Method II:
ΔBCD is a right triangle (CD is an altitude)
ΔCBE is a right triangle (BE is an altitude)
CD = BE (given)
BC = BC (common)
ΔBCD ≅ ΔCBE (HL)
Therefore, ∠DBC = ∠BCE

28. Draw MP so that MP⊥AB.
∠AMP = ∠MCN (corresponding angles)
PM = BN (BPMN is a rectangle)
Thus, ΔAPM ≅ ΔMNC (AS for right triangle)
Therefore,
AM = MC.



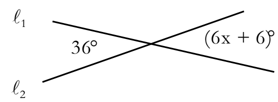
29. 12
BD = 5
$AD^2 = AB^2 - BD^2$
$= 13^2 - 5^2 = 12^2$
AD = 12

30. 60
$\text{area}(\Delta ABC) = \frac{1}{2}(10)(12) = 60$

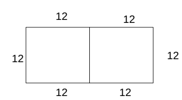
31. $\frac{120}{13}$
$\frac{1}{2}(13)(h) = 60$
$h = \frac{120}{13}$

Assessment Test

32. 5
Note vertical angles are equal. So, 36=6x+6.



33. 72
Look at the figure below. The overlapping part has a length 12×2=24. So, the perimeter = 96-24=72.



34. 18÷6=3 (width). The perimeter is 2(3+6)=18 ft.

35. The area is 20×30 - 10×14 = 600 - 140 = 460 in^2

36. The perimeter is 2×(20+30) + 2(20 - 6) = 100 + 28 = 128 in

37. What is the length of the larger rectangle? It is 40 cm. So, the perimeter of the larger rectangle = 2×(15 + 40) = 110 (cm)

38. The area of the larger =15×40 = 600(cm^2)

39. 5×25×2 = 250

40. The perimeter is 2×(25 + 5)×2 - 10 = 120 - 10 = 110 cm.

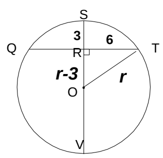
41. 5×25 = 125, 125 + 125 = 250, The overlapping part is 5×5=25, so 250-25 = 225.

42. 25 - 5 = 20, the perimeter is 4(10 + 5 + 10) = 100 cm.

43. From $OR^2 + RT^2 = OT^2$, we have $12^2 + 5^2 = OT^2$, OT = 13 (radius).

44. From $OR^2 + RT^2 = OT^2$, $4^2 + RT^2 = 8^2$, $RT = 4\sqrt{3}$, $QT = 2RT = 8\sqrt{3}$.

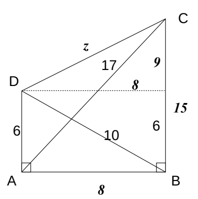
45. 7.5
See the figure. Let r be the radius. So, OR = *r*-3 (Why?). Use Pythagorean theorem, $r^2 = (r-3)^2 + 6^2$. $r^2 = r^2 - 6r + 9 + 36$. So, $6r = 45$. Thus, $r=7.5$



46. $x = \sqrt{10^2 - 6^2} = 8$

47. $y = \sqrt{17^2 - 8^2} = 15$

48. $z = \sqrt{(15-6)^2 + 8^2} \approx 12.04$



49. $64 \div 2 = 32$ in^2

50. $64 = 8 \times 8$, each side of the square is 8 in, the length of a rectangle is 8 in.

51. The width of a rectangle is $8 \div 2 = 4$, $2(8+4) = 24$ in

Assessment Test

52. B
$2(3\times5 + 5\times7 + 7\times3) = 142$

53. A
$4(3 + 5 + 7) = 60$

54. D

55. A

56. B

57. C
The width of the frame is 1 in.
The area of the A, a trapezoid, is $\frac{1}{2}(1)(6 + 8) = 7$

58. D
The width of the frame is 1 in.
The area of the A, a trapezoid, is $\frac{1}{2}(1)(10 + 8) = 9$

59. C
$8\times10 - 6\times8 = 80 - 48 = 32$
Note the area of the four trapezoid in the previous questions have a total area of $2(7 + 9) = 32$, coincides with this answer.

60. B
$40\times100 + \frac{1}{2}(40\times30) + \frac{1}{2}(30\times40)$
$= 4000 + 1200$
$= 5200$

61. C
$40 + 60 + 50 + 30 + 50 + 130 = 360$

62. C

63. B
$330\div2 = 165$

64. A
$4\times8\times5.5 = 4\times44 = 176$

65. B
$2(12 + 13) = 50$
50 ft = $\frac{50}{3}$ yd
$\frac{\frac{50}{3}}{5} = \frac{10}{3} = 3\frac{1}{3}$

66. D
$11\times13 = 143$

67. A
$\frac{base \times height}{2} = \frac{30\times83}{2} = 1245$

68. D
The trapezoid = $\frac{1}{2}(8)(10 + 13) = 92$

69. C
6 in = .5 ft
$9 \div .5 = 18$
$11 \div .5 = 22$
$18\times22 = 396$

70. B

Use Pythagorean theorem:

The hypotenuse = $\sqrt{10^2 + 24^2}$ =

$2\sqrt{5^2 + 12^2}$ = 2×13 = 26

71. D

2×4 = 8

20 + 8 = 28

40 + 8 = 48

The exterior rectangle is 28 by 48. The area of the side walk is

28×48 - 20×40 = 1344 - 800 = 544