Super Mathter

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Name: (First)____________(Last)____________

School: _________________ Grade:______

Perpendicular and Parallel Lines (1)

Question set [**1 - 2**]

For each pair of linear equations, determine if they are parallel, perpendicular, or neither.

1. $x - 3y = 4$
   $-2x + 6y = 0$

2. $2x + 3y = 0$
   $3x - 2y = 1$

Question set [**3 - 6**]

For each pair of linear equations, determine if they are parallel, perpendicular, or neither.

3. $y = 3x + 4$
   $2y = 6x + 9$

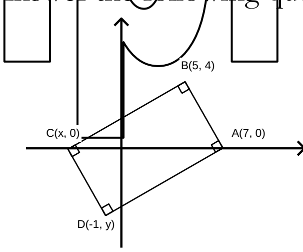
4. $y = 2x + 3$
   $y = -2x + 3$

5. $y = 3x$
   $y = \frac{1}{3}x$

6. $y = 0$
   $x = 3$

Question set [**7 - 12**]

As the figure below, ABCD is a rectangle. Answer the following questions.



7. What is the slope of (the line segment) AB?

8. What is the slope of BC?

9. What the X-coordinate (the value of x) of C?

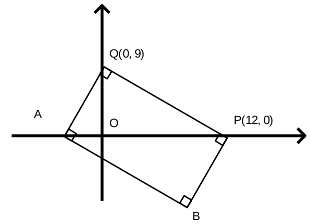
10. What is the slope of CD?

11. What is the Y-coordinate (the value of y) of D?

12. Compute the slope of AD using the result from the previous problem.

Question set [**13 - 18**]

As the diagram below, ABPQ is a rectangle. The coordinates of P and Q are given.



13. What is the slope of PQ?

14. Find the slope of AQ.

15. What is the coordinates of A?

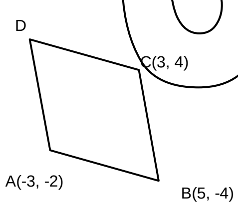
16. What is the area of ΔAPQ?

17. Find the slope of AB.

18. What is the slope of BP?

Question set [**19 - 22**]

As we know the two diagonals of a rhombus are perpendicular to each other. In the figure below, ABCD is a rhombus.



19. What are the relative coordinates of A to B?

20. Use the concept of relative coordinates, find the coordinates for D.

21. How can you verify if ABCD is a rhombus?

22. ABCD is rectangle. How do you verify it?

Question set [**23 - 30**]

Determine the linear equation for each of the following.

23. slope = -3, *y*-intercept = 6

24. slope = $\frac{1}{2}$, containing the point (-2, 3)

25. containing the points (-3, -2) and (4, -6)

26. containing two points: (1, 2), and (3, 4).

27. containing two points: (3, 0), and (0, 4).

28. containing the points (1, -2) and parallel to $3x + 2y = 1$.

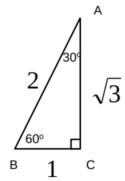
29. parallel to $y + 3x - 4 = 0$ and with the y-intercept = 6.

30. parallel to y-axis and passes through (3, 4).

Special Triangles

THEOREM A

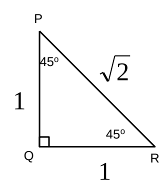
ΔABC is a special triangle: 30°-60°-90°. Then BC : AC : AB=1 : $\sqrt{3}$: 2.



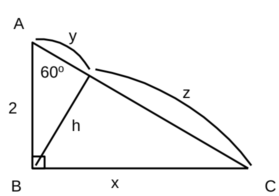
31. Prove the previous theorem.

THEOREM B

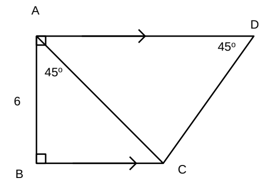
ΔPQR is a special triangle: 45°-45°-90°. Then PQ:QR:PR=1 : 1 : $\sqrt{2}$.



32. ΔABC is a right triangle with $\angle A$=60°. Find the value of x, y, z, and h.



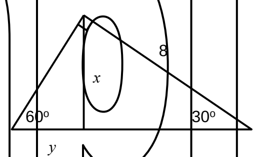
33. ΔABC is a right triangle with AB=6. Find the lengths of AC and AD. (Hint: ΔACD is a right isosceles.)



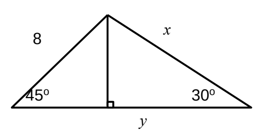
Question set [**34 - 39**]

Find the value of x and y in each of the following questions.

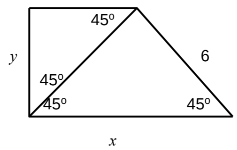
34.



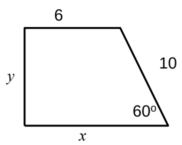
35.



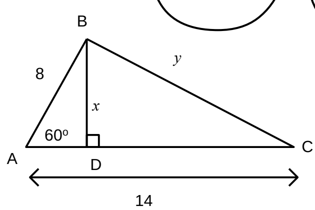
36.



37.

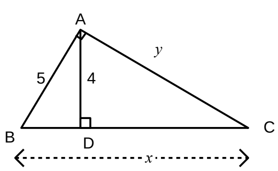


38.

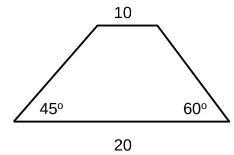


(Note: ∠ABC≠ 90°)

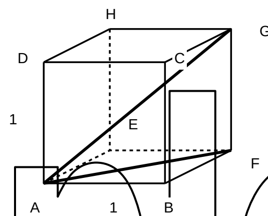
39. ∠BAC = 90°.



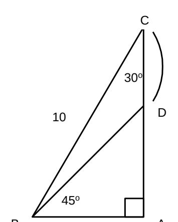
40. Find the height of the following trapezoid.



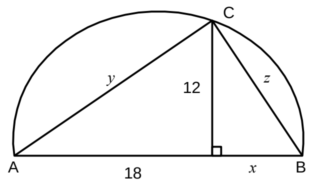
41. The figure shows a cube. Find the lengths of AF and AG.



42. ΔABC is a right triangle with ∠C=30°. BD is a segment in ΔABC with ∠ABD=45°. Find the length of CD.



43. ΔABC is a right triangle inscribed in a semicircle.
Find the value of x, y, z, and the area of the semicircle.



Radicals

Example A:
Simplify the following.

(a) $\sqrt[6]{\dfrac{x^{12}y^{24}}{64}}$

(b) $\sqrt[3]{\dfrac{(x-1)^6(y+5)^8}{216}}$

Solution:

(a) $\dfrac{x^2y^4}{2}$

(b) $\sqrt[3]{\dfrac{(x-1)^6(y+5)^8}{216}} = \dfrac{(x-1)^2(y+5)^4}{6}$

44. $\dfrac{\sqrt[3]{.5x^8y}}{\sqrt[3]{12.5x^2y^7}}$

45. $\dfrac{\sqrt[3]{5x^8y}}{\sqrt[3]{625x^2y^7}}$

46. $9\sqrt{50} - 6\sqrt{98} + 5\sqrt{32}$

47. $9\sqrt[3]{24} - 6\sqrt[3]{375} + 5\sqrt[3]{81}$

Example B:
Simplify: $(5\sqrt{8} + 2\sqrt{15})(5\sqrt{8} - 2\sqrt{15})$

Solution:
$(5\sqrt{8})^2 - (2\sqrt{15})^2$
$= 25(8) - 4(15)$
$= 200 - 60$
$= 140$

48. $(\sqrt{8} + \sqrt{15})(\sqrt{8} - \sqrt{15})$

49. $(\sqrt{a} + \sqrt{3b})(\sqrt{a} - \sqrt{3b})$

50. $(\sqrt{8} + 2\sqrt{15})(\sqrt{8} - 2\sqrt{15})$

51. $(\sqrt[6]{8} + \sqrt[6]{27})(\sqrt[6]{8} - \sqrt[6]{27})$

52. $(\sqrt[3]{2} + \sqrt[3]{5})(\sqrt[3]{4} - \sqrt[3]{10} + \sqrt[3]{25})$
(Hint: Use $(A+B)(A^2-AB+B^2)=A^3+B^3$)

53. $(\sqrt[3]{3} - \sqrt[3]{7})(\sqrt[3]{9} + \sqrt[3]{21} + \sqrt[3]{49})$

54. $(\sqrt{5} + 3)^2$

55. $(3\sqrt{5} - 7)^2$

56. $(3\sqrt{5} - \sqrt{7})^2$

Math Challenge

57. Let A, M, and C be non-negative integers such that A + M + C = 12. What is the maximum value of AMC + AM + MC + CA?

Negative Exponents

Example C:
Simplify each of the following.

(a) $t^2 \cdot 3t^4/(4t^3) =$

(b) $\frac{-4p^4 \cdot 3p^6}{3p^3 \cdot 4p^5} =$

Solution:

(a) $t^2 \cdot 3t^4/(4t^3) = \frac{t^2 \cdot 3t^4}{4t^3} = \frac{3t^6}{4t^3} = \frac{3t^3}{4}$

(b) $\frac{-4p^4 \cdot 3p^6}{3p^3 \cdot 4p^5} = \frac{-p^{10}}{p^8} = -p^2$

Question set [**58 - 67**]
Simplify the following.

58. $\frac{(-x)(-2x^2)(-3x^3)}{(-4x^4)(-5x^5)} =$

59. $\frac{(w^2)^6}{(w^3w^2)^4} =$

60. $\frac{-6x^3y^2}{-4x^2y^6} =$

61. $(-2a^3)(5ab^2)/(-3a^4b) =$

62. $(\frac{-3x^3}{y^4})^2(\frac{x^7}{6y^5})^3 =$

63. $\frac{(a^2)^2}{(a^3)^3} =$

64. $\frac{-2x^2y}{4x^3y^3} =$

65. $\frac{5s^2t^3}{3s^4} =$

66. $(5ax)(3ax^3)/(2a^2x^5)=$

67. $\frac{(s^3)^8}{(s^2)^5}=$

Answer Key

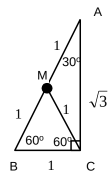
Perpendicular and Parallel Lines (1)

1. Since 1:-3 = -2:6, they are parallel.
2. Since the slope of the first line is $-\frac{2}{3}$ and the second one has slope = $\frac{3}{2}$, thus they are perpendicular.
3. Both have the same slope 3, so they are parallel.
4. They are neither parallel nor perpendicular.
5. They are neither parallel nor perpendicular.
6. They are perpendicular since the first one is horizontal and the second one is vertical.
7. $\text{slope(AB)} = \frac{\Delta y}{\Delta x} = \frac{4-0}{5-7} = -2$
8. The slope of BC is $\frac{1}{2}$ according to the theorem.
9. $\frac{1}{2} = \text{slope(BC)} = \frac{0-4}{x-5} \Rightarrow x-5 = -8 \Rightarrow x = -3$.
10. CD is parallel to AB, thus its slope is equal to that of AB, which is -2.
11. $-2 = \text{slope(CD)} = \frac{y-0}{-1-(-3)} \Rightarrow y = -4$.
12. $\text{slope(AD)} = \frac{\Delta y}{\Delta x} = \frac{-4}{-1-7} = \frac{1}{2}$
13. The slope of PQ = $\frac{-9}{12} = \frac{-3}{4}$
14. AQ is orthogonal to PQ, its slope should be $\frac{4}{3}$.
15. Since the slope of AQ is $\frac{4}{3}$, OQ/AO=4/3 $\Rightarrow$ 9/AO=4/3 $\Rightarrow$ AO=27/4.
16. $\frac{1}{2}$(base)×(height) = $\frac{1}{2}$(12 + 27/4)×9 = $\frac{1}{8}$(75×9) = $84\frac{3}{8}$
17. Since AB is parallel to PQ, its slope should be $-\frac{3}{4}$,
18. BP//AQ, so the slope is the same: $\frac{4}{3}$.
19. A - B = (-3, -2) - (5, -4) = (-8, 2)
20. D = (3, 4) + relative coordinates of A to B = (3, 4) + (-8, 2) = (-5, 6)
21. AB = $2\sqrt{17}$ = BC
22. slope(AB)×slope(BC) = -1, which means AB⊥BC.
23. $y = -3x + 6$
24. First of all, we know the equation of the line should be something like $y = \frac{1}{2}x + b$ since the line passes (-2, 3), we conclude that b=4.
25. First we need to decide the slope of the line, using the slope formula, we know slope = $\frac{-4}{7}$. Thus, the equation is something like $y = \frac{-4}{7}x + b$. Since the line passes through (-3, -2), b= $\frac{-26}{7}$.
26. $y = x + 1$
27. These two points are the x- and y-intercepts. The equation is $\frac{1}{3}x + \frac{1}{4}y = 1$.
28. Being parallel to $3x + 2y = 1$, so the equation of the line should be something like $3x+2y = c$. Passing through the point (1, -2 leads us to solve for c, thus, c = -1.
29. The equation should be as $y + 3x + c$. Since its y-intercept = 6, it must passes through (0, 6). Thus, 6 + c = 0 $\Rightarrow$ c = -6. So, the equation should be $y + 3x - 6 = 0$.

30. $x = 3$

Special Triangles

31. Draw an auxiliary line CM so that $\angle BCM = 60°$. ΔBCM is an equilateral and ΔMAC is an isosceles with AM = CM. (Why?) Thus, BM = MA = 1, and AB = 2. Using Pythagorean theorem AC = $\sqrt{3}$



32. $x = 2\sqrt{3}$, $y = 1$, $h = \sqrt{3}$, $z = 3$

33. BC = 6, AC = $6\sqrt{2}$, AD = 12

34. $x = 4$

$y = \frac{4}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$

35. $x = 2(\frac{8}{\sqrt{2}}) = 8\sqrt{2}$

$y = 4\sqrt{6}$

36. $x = 6\sqrt{2}$

$y = 3\sqrt{2}$

37. $x = 6 + 5 = 11$

$y = 5\sqrt{3}$

38. $x = 4\sqrt{3}$

AD = 4, CD = 10

$y = \sqrt{CD^2 + BD^2} = \sqrt{148} = 2\sqrt{37}$

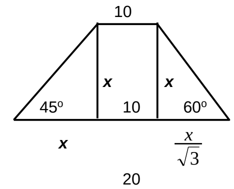
39. BD = 3

$3x = 5^2 \Rightarrow x = \frac{25}{3}$

$y^2 = x\cdot(\text{CD}) = \frac{25}{3}(\frac{25}{3} - 3) = \frac{25\cdot 16}{9} \Rightarrow y = \frac{20}{3}$

40. Let x be the height, then the base is $20 = x + 10 + \frac{x}{\sqrt{3}}$

$\Rightarrow x = \frac{10\sqrt{3}}{\sqrt{3}+1} = 5\sqrt{3}(\sqrt{3} - 1) = 15 - 5\sqrt{3}$



41. AF= $\sqrt{2}$, AG= $\sqrt{3}$

42. AB=5, AC=$5\sqrt{3}$, AD=5 $\Rightarrow$ CD=$5\sqrt{3}$ -5 = $5(\sqrt{3}$ -1)

43. Since $18x = 144$, $x = 8$. Use Pythagorean theorem, $y = 6\sqrt{13}$, $z = 4\sqrt{13}$. The diameter is 18+8 = 26, the radius is 13, the area of the circle is 169π.

Radicals

44. $\frac{\sqrt{9x^7y^5}}{\sqrt{16x^3y^9}} = \sqrt{\frac{9x^7y^5}{16x^3y^9}} = \frac{3x^2}{4y^2}$

45. $\frac{\sqrt[3]{.5x^8y}}{\sqrt[3]{12.5x^2y^7}} = \sqrt[3]{\frac{5x^8y}{125x^2y^7}} = \frac{x^3}{5y^3}$

46. $9\sqrt{50} - 6\sqrt{98} + 5\sqrt{32} = 45\sqrt{2} - 42\sqrt{2} + 20\sqrt{2} = 23\sqrt{2}$

47. $9\sqrt[3]{24} - 6\sqrt[3]{375} + 5\sqrt[3]{81} = 18\sqrt[3]{3} - 30\sqrt[3]{3} + 15\sqrt[3]{3} = 3\sqrt[3]{3}$

48. $(\sqrt{8} + \sqrt{15})(\sqrt{8} - \sqrt{15}) = \sqrt{8}^2 - \sqrt{15}^2 = 8 - 15 = -7$

49. $(\sqrt{a} + \sqrt{3b})(\sqrt{a} - \sqrt{3b}) = a - 3b$

50. $(\sqrt{8} + 2\sqrt{15})(\sqrt{8} - 2\sqrt{15}) = 8 - 4(15) = -52$

51. $(\sqrt[6]{8} + \sqrt[6]{27})(\sqrt[6]{8} - \sqrt[6]{27}) = (\sqrt[6]{2^6} - \sqrt[6]{3^6}) = 2 - 3 = -1$

52. $(\sqrt[3]{2} + \sqrt[3]{5})(\sqrt[3]{4} - \sqrt[3]{10} + \sqrt[3]{25}) = \sqrt[3]{2}^3 + \sqrt[3]{5}^3 = 2 - 5 = -3$

53. $(\sqrt[3]{3} - \sqrt[3]{7})(\sqrt[3]{9} + \sqrt[3]{21} + \sqrt[3]{49}) = \sqrt[3]{3}^3 - \sqrt[3]{7}^3 = 3 - 7 = -4$

54. $(\sqrt{5} + 3)^2 = 5 + 6\sqrt{5} + 9 = 15 + 6\sqrt{5}$

55. $(3\sqrt{5} - 7)^2 = 9(5) - 42\sqrt{5} + 49 = 94 - 42\sqrt{5}$

56. $(3\sqrt{5} - \sqrt{7})^2 = 9(5) - 3\sqrt{35} + 7 = 52 - 3\sqrt{35}$

Math Challenge

57. 112
When A=M=C = 4, it reaches the maximum, so the answer is 64 + 3×16 = 112.

Negative Exponents

58. $(-x)(-2x^2)(-3x^3)/\{(-4x^4)(-5x^5)\} = \frac{3}{-10x^3}$

59. $(w^2)^6 / (w^3 \cdot w^2)^4 = \frac{1}{w^8}$

60. $-6x^3 y^2 / (-4x^2 y^6) = \frac{3x}{2y^4}$

61. $(-2a^3)(5ab^2)/(-3a^4b) = \frac{10b}{3}$

62. $(\frac{-3x^3}{y^4})^2 (\frac{x^7}{6y^5})^3 = \frac{9x^6}{y^8} \frac{x^{21}}{6^3 y^{15}} = \frac{9x^{27}}{216y^{23}}$

63. $(a^2)^2/(a^3)^3 = \frac{1}{a^5}$

64. $(-2x^2 y)/(4x^3y^3) = -\frac{1}{2xy^2}$

65. $5s^2 \cdot t^3 / (3s^4) = \frac{5t^3}{3s^2}$

66. $(5ax) \times (3ax^3)/(2a^2 x^5) = \frac{15}{2x}$

67. $(s^3)^8 / (s^2)^5 = s^{14}$

PreCal